CONTEMPORARY ECONOMIC POLICY

THE 80% PENSION FUNDING TARGET, HIGH ASSUMED RETURNS, AND GENERATIONAL INEQUITY

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Generational inequity in pension funding is highly sensitive to the lax policies of 80% funding targets and high assumed returns to investment. I develop a simple, powerful relationship between steady-state (SS) inequity in contributions—the percent of extra contributions to fund prior cohorts—and the SS unfunded ratio. I then show how the SS unfunded ratio is governed by x*% funding targets and the gap between assumed and true returns. The SS degree of inequity is over 60% under an 80% funding target and over 50% with a one-point gap between assumed and true returns.* (*JEL* H75)

I. INTRODUCTION

Employer costs for state and local pensions have doubled as a percent of payroll in the last decade, from 7.7% in 2004 to 15.4% in 2015.[1] The rise largely reflects payments on unfunded liabilities, as the aggregate funded ratio (assets/liabilities) has dropped from 87% in 2004 to 73% in 2014.[2] The lack of political will to fully address these liabilities has been abetted by two ideas: (1) full funding is unnecessary for "sustainability"—80% is good enough; and (2) high assumed returns on investment (e.g., 8%) are also sustainable, even if returns have fallen short of late. The purpose of this study is to formally analyze two policies—an "*x*% funding policy" and a "high assumed return policy." Using the simple mathematics of ordinary difference equations, I focus on the existence, stability, and characteristics of steady state (SS)—the condition under which the funded ratio is constant. Of particular policy interest is the SS degree of generational inequity in contributions.

What is the empirical and policy motivation for formally exploring these questions? To be clear, there are almost no plans that explicitly build a target funded ratio of less than 100% into their funding formula, so the motivation here is not to analyze such funding formulas actually in use.[3] Instead, one motivation—a modest one—is to simply show how such a target ratio would play out, were it to be incorporated into an otherwise standard funding formula. More importantly, quite a number of plans that target full funding have fallen well short of meeting that target for many years, and have often used the 80% rationale to avoid taking corrective action.[4] Two of the main reasons for the shortfalls (Costrell 2016; Munnell, Aubry, and Cafarelli 2015) are (1) the failure to make actuarially required contributions and (2) overly optimistic actuarial assumptions—notably the assumed investment return—which artificially depress required contributions in the short run. The incentive to rectify such shortfalls depends

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1. These data are from the Bureau of Labor Statistics National Compensation Survey, Employer Cost for Employee Compensation. The Boston College Public Plans Database gives very similar figures.

2. Boston College Public Plans Database (2016).

3. The exceptions here are the five state systems of Illinois. Under Illinois statute, the 2045 target is 90%. In addition to the target itself, the method by which that target is built into the funding formula is quite nonstandard—unlike the method modeled below. There are other states that have built 80% triggers into their COLAs.

4. See the statements of New Jersey and Connecticut policy-makers cited in American Academy of Actuaries (2012).

ABBREVIATIONS

ARC: Annual Required Contribution
GFOA: Government Finance Officers Association
OLG: Overlapping Generation
SS: Steady State
UAL: Unfunded Accrued Liability

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critically on the debate over whether 80% (or some similar figure) is "good enough."

With regard to (1), the "*x*% funding policy" model below may be interpreted as a formal analysis of underfunding "as if" a funded ratio of less than 100% had been built into the formula. The Boston College Public Plans Database (2016) reports that, on average, employers have left unpaid 15% or more of the contributions designed to reach full funding, since 2010. Although the policies that yield this result are informal, it may be useful to model them "as if" the target funded ratio was *x*%. With regard to (2), the "high assumed returns" model below is directly pertinent to existing practice. In the aftermath of the 2007–2009 market crash, many funds reduced their assumed returns from about 8%, but not by much (0.27%, on average, Biggs 2015), despite widespread expectations in the investment community of lower future returns (Dobbs et al. 2016). Both policies—"*x*% funding" and "high assumed returns"—have strong implications for generational inequity by reducing contributions by and for the current cohort, to be made up by future cohorts, to a degree that can be formalized.

Before delving into the analysis, I will first review the background and literature on the "80% standard" and the rationale for my formal analytical approach. The new results from that approach will be summarized below, but the main policy implication can be stated here: small departures of the policy variables from sound practice—target funded ratios below 100% and high assumed returns—result in large SS underfunding and generational inequity in contributions. To reduce generational inequity, it will be necessary to hew closely to the 100% funding target and to significantly mark down assumed returns.

II. BACKGROUND AND PREVIOUS LITERATURE ON THE "80%" STANDARD

Continuing public pension funding shortfalls, years after the market crash of 2007–2009, have given circulation to the notion that "full funding" is unnecessary for a "healthy" pension plan. A particularly common form of this idea is the "80%" standard, according to which professional opinion allegedly views an 80% funded ratio as the threshold for a sound plan (see, e.g., Appendix of American Academy of Actuaries 2012). The origins of this standard are obscure, and possibly mythical (see Miller 2012; and reply by Brainard and Zorn 2012). Indeed, there is now a website, the "80 Percent Pension Funding Hall of Shame" (Campbell 2014) and associated database, devoted to exposing those public officials and pension industry participants who perpetuate this "standard." That said, the claim does raise questions of the precise sense in which a certain minimum funded ratio indicates a fund's "health," or, more specifically, its "sustainability." Can target ratios below 100% be "sustainable," and, if so, how far below 100%, and what are the consequences?

Some observers of the public pension industry have astutely clarified the main issues with the "80% standard." Notably, Miller (2012) makes two key distinctions. First, he points out that it is one thing to hit 80% funding at the bottom of the market, while still averaging 100% over the cycle by, for example, reaching 125% at the peak.[5] It is quite another matter to average well below 100% indefinitely. The American Academy of Actuaries (2012) makes a similar (though not identical) distinction between a 100% target ratio for actuarial funding (which they recommend) and an 80% snapshot at a point in time, which may or may not be on track to reach the 100% target.

The second distinction Miller makes concerns the criteria for plan health: sustainability versus generational equity.[6] Even if the plan averages below 100% indefinitely, it may still be "sustainable" with contribution rates that cover amortization of the unfunded accrued liability (UAL). However, this runs afoul of generational equity, with "current taxpayers supporting retirees who didn't ever work for them" and/or the employees paying down previous cohorts' unfunded liabilities.

Still, confusion remains in publications of public pension industry spokesmen (let alone the general public and policy-makers). The Government Finance Officers Association (GFOA 2009) specifies a target funded ratio of "100% or more" as the first required practice for "sustainable funding." As Miller has clarified, this is actually a requirement for generational equity, rather than sustainability. However, the water was further muddied by spokesmen for the National Association of State Retirement Administrators and the prominent actuarial firm Gabriel, Roeder, Smith & Company. In a joint statement of these

5. However, as Miller points out, funded ratios above 100% create a political problem, tempting lawmakers to take pension holidays or enhance benefits.

6. American Academy of Actuaries (2012) also alludes to this distinction.

two entities (replying, in part, to Miller), Brainard and Zorn (2012) reiterate GFOA's recommendation of 100% target funding for sustainability, but then suggest it is not critical to ever reach that target: "many pension plans remain underfunded for decades without causing fiscal stress for the plan sponsor or requiring benefits to be reduced." For them, the "critical factor" in evaluating plan health is whether or not the required contributions are so high as to create "fiscal stress." More to the point, their statement appears to ratify an *x%* standard as a rationale for inaction in addressing the sources of underfunding, whether it is failure to meet required contributions or overly optimistic assumed investment returns.

III. RATIONALE FOR THE FORMAL ANALYTICAL APPROACH AND SUMMARY OF RESULTS

This study brings a simple analytical model to bear on these issues. Although recently developed simulation models incorporate risk and other features, one can obtain insight into the questions posed here by a model stripped to its essentials (e.g., risk-free investment) and solvable from a simple ordinary difference equation. For example, the notion of "sustainability"—a term that is often used without precise definition—is logically defined in such a model as stability of a SS, at an equilibrium funded ratio and corresponding contribution rate.

The first benefit of such a model is to sort out formally who is right and who is wrong in the commentaries discussed above. Miller is right: SS funded ratios below 100% are "sustainable," if generationally inequitable, in a precise mathematical sense. There is a continuum of stable ("sustainable") steady states. There is no *x%* minimum for the SS funded ratio in this model, other than the 0% criterion of solvency. Low SS funded ratios correspond to high SS contribution rates, exceeding normal cost (the cost of prefunding each cohort's benefits). The problem with *x%* funding, therefore, is generational inequity, not "sustainability." The model helps us better understand how this result obtains, showing that the system's stability is unrelated to the funding target, and identifying instead the features of an actuarial funding formula that are required to secure stability.

The model also generates additional results not previously articulated. My model generates a simple, powerful relationship between the SS unfunded ratio and a meaningful measure of generational inequity in contributions. The model also provides deeper understanding of the SS funded ratio itself. I show that if a system *targets* an *x%* ratio in its amortization formula, the SS ratio will be lower yet. That is because contributions at the *x%* target include no amortization, and normal cost alone is insufficient to sustain that target. Conversely—and more pertinently for actual policy practices—to achieve any given SS funded ratio, one must set a higher target ratio for amortization purposes. If the true goal is 80% funding in SS, the target for amortization purposes must be much closer to 100%.

As a corollary of this result, to merely achieve solvency (0% SS ratio) the target ratio must be set at a positive floor. To take that extreme case, if the target ratio is set at that lower bound, the resulting SS reproduces a pay-go system, under the guise of a prefunding formula, with amortization payments making up the difference between the pay-go rate and the normal cost of prefunding benefits. More generally, the math simplifies nicely to allow easy calibration of the relationship between the SS ratio and the amortization target, and to demonstrate its sensitivity.

Finally, I model the policy of high assumed returns,[7] which inflates the funded ratio. The model shows that even by the inflated ratio the system is underfunded in SS, let alone by the true ratio. Moreover, the measured SS degree of generational inequity will exceed that based on the measured SS unfunded ratio. The main policy takeaway is that small deviations from the target of full funding or of the assumed return from true returns generate large degrees of generational inequity in contributions.

IV. THE BASIC MODEL

Consider a population of public employees in a traditional defined benefit plan, such as a "final-average-salary" plan, where the initial annuity equals years of service × "multiplier" × final average salary. For example, with a multiplier of

7. Virtually the entire finance economics profession (led by Brown and Wilcox 2009 and Novy-Marx and Rauh 2009, 2011) has argued that the assumed return used by public fund actuaries is too high for discounting liabilities. The debate distinguishes between the accounting rate used for reporting liabilities—which should be the risk-free rate—and the rate used for funding purposes. In this study, my focus is the funding system, so I do not distinguish between the assumed return and the liability discount rate used for the funding calculation.

2.0%, after serving 30 years one may receive 60% of final average salary for life, plus any COLAs.

The basic pension funding math can be set out with the following notation:

W_t = payroll in period t, the product of salary and number of employees

$G = W_t/W_{t-1} = 1 +$ growth rate of payroll

$R = 1 +$ return on investment (assumed to be certain)

c_t = contribution to pension fund, as fraction of payroll, in period t (joint: employer and employee), to be specified further below

c^n = "normal cost," fraction of payroll to prefund pension

c^p = "pay-go cost," fraction of payroll to pay annual pension benefits

A_t = assets in pension fund, at end of period t

L_t = accrued liabilities of the pension fund, at end of period t

$f_t = A_t/L_t$, funded ratio at end of period t

Note that R is taken as constant, as my focus here is not the role of market fluctuations. In addition, G, c^n, and c^p are taken as constants.[8] In doing so, I am assuming the population is in SS (a "mature" population, to use the actuarial term), so that I can focus on the behavior of funding, in and out of SS, to determine its "sustainability."

The timing sequence of the model is this: during period t, employees receive total payroll W_t from taxpayers; employees and taxpayers jointly contribute c_tW_t to the fund; retirees receive pension benefits of c^pW_t from the fund; and the fund earns returns R on the end-of-previous-period balance, A_{t-1}. Thus, the fund evolves as:

$$(1) \qquad A_t = RA_{t-1} + \left(c_t - c^p\right) W_t.$$

The accrued liability at time t is the present value of accrued benefits to be paid in the future, which, for a mature population, grows in step with payroll, at rate G. It can also be expressed as the prior liability grown by R (since the previously accrued benefits are one year closer), plus the current accrual of new benefits—normal cost—less the benefits paid out[9]:

$$(2) \qquad L_t = GL_{t-1} = RL_{t-1} + \left(c^n - c^p\right) W_t.$$

Substituting $L_{t-1} = L_t/G$ on the RHS and solving, we have:

$$(3) \qquad L_t = \left[G/\left(R-G\right)\right] \left(c^p - c^n\right) W_t.$$

This SS relationship between accrued liabilities and payroll will be quite useful below. It represents the difference between the present value of all future benefit payments and all future liability accruals (normal costs).[10] These are, respectively, a fraction c^p and c^n of the present value of future payroll, $[G/(R-G)]W_t$. Note also that since accrued liabilities are positive, the condition $R > G$ (the relevant case, as discussed below) implies $c^n < c^p$. It is cheaper to prefund one's own pension than to pay for previous cohorts, if the return to investment R is high and/or G is low (so that previous cohorts are not much smaller than one's own).

A. A General Result on Generational Inequity

The system's dynamic behavior will depend on the funding formula governing contributions, c_t, but even before specifying that formula, we can derive the SS relationship between the contribution rate and the funded ratio. Since L_t grows at rate G, so must A_t, for the funded ratio f_t to be constant at its SS value, call it f^*. From Equation (1), this means the SS contribution rate, c^*, must satisfy:

$$(4) \qquad \begin{aligned} G = A_t/A_{t-1} &= R + \left(c^* - c^p\right) W_t/A_{t-1} \\ &= R + \left(c^* - c^p\right) W_t/\left(f^* L_{t-1}\right) \\ &= R + \left(c^* - c^p\right) W_t/\left(f^* L_t/G\right). \end{aligned}$$

Substituting from Equation (3) and solving, we have a fairly general result:

$$(5) \qquad c^* = c^n + \left(c^p - c^n\right)\left(1 - f^*\right).$$

The first term on the RHS, the normal cost, is the cost of prefunding any cohort's future benefits. The second term represents each generation's contribution over and above what is required

8. As a special case, one may consider a simple two-period OLG model (Diamond 1965; Samuelson 1958). Each generation of public employees works for one period (e.g., 30 years) and then retires (also for 30 years). In this simple model, let p represent one's pension, as a fraction of prior salary. Then $c^n = p/R$, $c^p = p/G$, and $L_t = pW_t/R$, where R and G are now understood to be (say) 30-year compound rates.

9. The precise measurement of normal cost varies with actuarial convention in the multiperiod case. That is because the partition of the present value of all future benefits between those that have already accrued and those yet to be accrued depends on conventions regarding when to recognize projected service and wage growth based on current and prior service and wages. To fix ideas, one may consider entry age normal, the current standard under the Governmental Accounting Standards Board, but the analysis below pertains to other methods as well.

10. This follows from the basic identity given in the previous note that the present value of all future benefit payments equals the present value of all benefits yet to be accrued and the present value of all benefits previously accrued, but not yet paid out. The latter term is the accrued liability.

to pay for its own benefits. More specifically, $(c^p - c^n)$ represents the *potential* extra burden imposed on the current cohort to fund the benefits of prior cohorts. The fraction of that burden paid in SS, $(1 - f^*)$, may be considered a measure of generational inequity. At full funding $(f^* = 1)$, that burden is zero; at the opposite extreme $(f^* = 0)$, the full burden is borne, so $c^* = c^p$. That is, we have a strikingly simple result for generational inequity: *the extra annual burden borne by each cohort (a flow variable) is the SS unfunded ratio (a ratio of stock variables) times the difference between the pay-go rate and the normal cost.*

In the remainder of this article, I will examine the determinants of the SS funded ratio f^* (and, hence, the degree of generational inequity, through the result just established), under the two policies in question: x% funding and high assumed returns. First, however, I examine the issue of stability, since SS is of little interest unless it is stable. Indeed, going back to the debate over these policies, our understanding of the term "sustainable" is that a meaningful SS exists, and it is stable.

B. Stability

The stability of the system is governed by the contribution policy underlying the sequence of contribution rates c_t. Consider briefly the case of exogenous c, to see why it must be endogenized, in accord with standard actuarial practice. If c_t is fixed at c, the SS funded ratio is $f^* = (c - c^p)/(c^n - c^p)$ and the stability condition is $R < G$ (see Appendix). Specifically, as is well established in the overlapping generation (OLG) literature, if $R < G$, $c^p < c^n$, and pay-go $(c = c^p)$ is sustainable[11]: contributions cover pension payments with no need to hold any balance in the fund $(f^* = 0)$. If the funded ratio is positive to begin with, it will shrink toward zero. Any higher contribution rate, $c > c^p$, is inefficient, and leads to unnecessary asset accumulation $(f^* > 0)$.

The problem is that typically $R > G$. Indeed, this assumption has guided pension policy for the last few decades.[12] Certainly once the baby boomers entered the workforce, actuaries and economists reading the demography reports knew the baby bust would require moving from pay-go to prefunding (generally starting in the vicinity of 1980). The actuarially assumed rate of return on public pension funds currently averages 7.7%, in the Public Plans Data of the Boston College Center for Retirement Research,[13] exceeding the average assumed rate of payroll growth of about 3.7%.[14] With $R > G$, the system is unstable for constant c. The funded ratio would veer off to plus/minus infinity, as $f_0 >(<) f^*$. Specifically, the normal cost is less than pay-go, but if contributions were simply set to c^n (so $f^* = 1$), the system would collapse for any starting balance $f_0 < 1$.

Contributions need to be endogenous to address the stability problem when $R > G$. Standard actuarial funding formulas do so through contributions that annually amortize some portion s of unfunded liabilities, after paying normal cost. In this way, contributions are adjusted over time, with the intention of steering the funded ratio toward one. Moreover, the same logic holds if the target funded ratio is set below one, as specified in the section below. Specifically (see Appendix), the stability condition becomes $s > R - G$. Under this condition, amortization payments will exceed the growth-adjusted interest on the (target) unfunded liability. Importantly, the stability condition is independent of the target funded ratio. This contrasts with the suggestion that an 80% funded ratio (or some other figure) is tied to the "sustainability" of the system. The issue is generational inequity, not "sustainability."

V. TARGET FUNDED RATIO < 100%

Consider the following stylized model of actuarially required contributions, as a fraction of payroll:

$$c_t = c^n + s\left(f^\circ L_{t-1} - A_{t-1}\right)/W_t. \qquad (6)$$

The second term is the amortization payment. Normally it is a fraction, s, of the difference

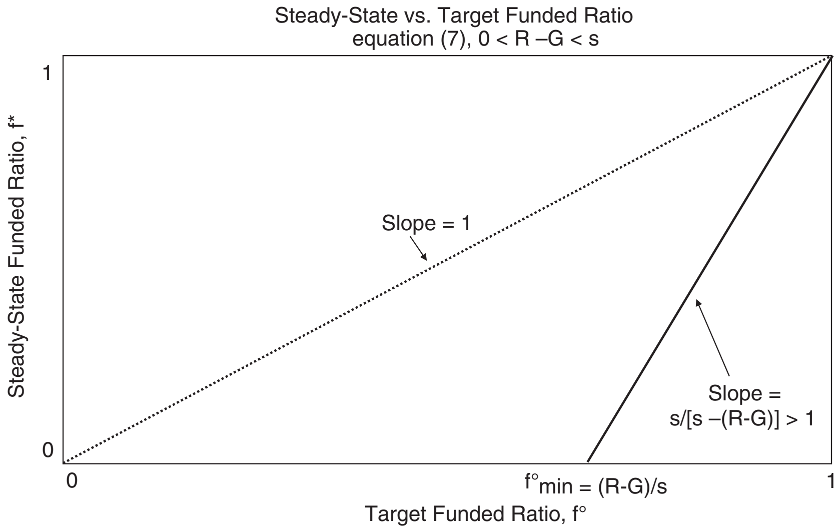
11. Conversely to the discussion below (3), it is cheaper to pay for previous cohorts' pension than to prefund one's own, if the previous cohort is small (G is high) and/or the return to investment R is low. Of course, this literally requires growth at rate G forever, so that no "last generation" gets stuck holding the bag.

12. It is also a standard result from growth theory, although Piketty (2014) created something of a stir by finding this result in the historical record. See Mankiw (2015).

13. This is the unweighted FY13 estimate, as reported for 145 of the 150 state and local plans covered; the asset-weighted estimate is almost identical. It is also equal to the national average for state and local pensions, from the Census of Governments, as reported by the Boston College Retirement Center (accessed January 8, 2016). The assumed rate has been reduced in recent years for a number of plans, but the national average from the Census of Government has not dropped much, from 7.98% in 2003 to 7.64% in 2013.

14. This is both the weighted and unweighted FY13 estimate, as reported for 42 of the 150 plans covered.

FIGURE 1
Steady-State Versus Target Funded Ratio



between liabilities and assets (as most recently measured), but I have slightly generalized it here to allow for an x% target funded ratio $f°$.

There are two potential rationales for this formulation (even though there are no plans that formally embed a target ratio below 100% in this fashion[15]). First, if plans were to formalize a target below 100% within a standard formula, this would appear to be the natural way to do it (as I will show, however, for this to work, $f°$ would have to be set higher than the true target). A second rationale pertains to an observed practice, where the funded ratio persists at less than 100% due to shortfalls in contributions, but authorities take no corrective action, falling back on the "80%" rule for a rationale. In that case, Equation (6) may be thought of as an implicit funding formula, modeling contributions "as if" $f° < 1$ were embedded in it.

A. Steady State

As shown in the Appendix, the SS funded ratio is:

$$(7) \quad f^* = \left[sf° - (R - G)\right] / [s - (R - G)].$$

If the target is full funding ($f° = 1$), then the SS will indeed be full funding ($f^*=1$). However, if the target is for less than full funding, the SS ratio will be lower yet.[16] That is, if the target is set to 80% funding, the actual SS will be lower than 80%. The reason is that once we reach 80%, amortization will be zero, so contributions will simply cover normal cost ($c_t = c^n$), which is not enough to sustain SS unless we are at full funding. Indeed, the *meaning* of contributing normal cost is that each cohort has prefunded its benefits, so that $f = 1$. If $f_{t-1} = f° < 1$, so that $c_t = c^n$ (by Equation (6)), the funded ratio will fall (and contributions will rise) until f approaches the SS given in Equation (7), $f^* < f° < 1$.

Consequently, if policy-makers' true target is 80%, the target funded ratio for amortization purposes ($f°$) must be set higher than 80%. Indeed, we would expect that policy-makers informally following an 80% rule would not allow f to fall much below 80%, but would, instead ultimately act "as if" the amortization target $f°$ was high enough to yield f^* of 80%, as determined by Equation (7).

Figure 1 illustrates the relationship between the amortization target, $f°$, and the SS f^*, for a given value of s. The relationship lies below the 45-degree line. This can be read from the

15. The Illinois plans that formalize a 90% target do so in a non-standard formula that works backward from a fixed (but distant) target year. According to the actuaries for the Illinois state plans (who dub the state's funding formula as "Illinois math"), the funding method effectively sets a lower target than the standard method. (Teachers' Retirement System of the State of Illinois 2016 and earlier).

16. From Equation (7), $f^*[s - (R - G)] = [sf° - (R - G)] < f°[s - (R - G)]$ for $f° < 1$, since $R > G$, so $f^* < f°$.

amortization target $f°$ to the consequent SS, which is lower, or from the SS one seeks to the required target for amortization, which is higher.

The horizontal intercept in Figure 1 is $f°_{\min} = (R - G)/s > 0$, from Equation (7). That is, if the true target is 0% (i.e., barely solvent), the target ratio for amortization must be set somewhat greater than zero. Thus, there is a positive floor for the amortization target ratio, below which insolvency will ensue, so there is a sense in which one could say that a minimum $x\%$ target ratio is necessary, but it is the amortization target, rather than the true target.

As shown earlier, in Equation (5), the SS contribution rate c^* will include a fraction of the extra burden to fund the benefits of prior cohorts, $(c^p - c^n)$, and that fraction is the SS unfunded ratio, $(1 - f^*)$. The model shows how that fraction is related to the amortization target $f°$. Specifically, using Equation (7) in Equation (5), the SS contribution rate is:

$$(8)\quad \begin{aligned} c^* &= c^n + (c^p - c^n)\left(1 - f^*\right) \\ &= c^n + (c^p - c^n)\, s\left(1 - f°\right) / [s - (R - G)] \\ &> c^n + (c^p - c^n)\left(1 - f°\right). \end{aligned}$$

As the target $f°$ drops from 100% to, say, 80%, our measure of generational inequity rises from zero to something greater than 20% (indeed, much greater, as shown below) of the extra burden $(c^p - c^n)$, since $(1 - f^*) > (1 - f°)$. In the limiting case, as the amortization target reaches $f°_{\min}$, so $f^* = 0$, the full extra burden of funding prior cohorts is borne: $c^* = c^p$. The form that extra burden takes is the amortization payment. That is, if we set the amortization target as low as possible, consistent with SS solvency, the amortization payments suffice to top up the normal costs to the pay-go rate. The only difference between this system and the simple pay-go system is that with $R > G$, the system with a constant contribution rate is unstable, while in this system, with $s > (R - G)$, the amortization payments will adjust outside of SS to provide stable convergence.

B. Standard N*-Period Amortization and Calibration of Magnitudes*

To get more specific (and to calibrate magnitudes), we specify the amortization rate, s, using a standard actuarial formula. The most common formula is "level percent" of payroll. Amortization payments are set to grow in step with payroll, at rate G and to pay off the unfunded liability in N years (usually 30).[17] Under the "open" version—commonly used—the N-year horizon is renewed every year. Under this formula,[18]

$$(9)\qquad s = (R - G) \,/\, \left[1 - (G/R)^N\right].$$

This will always satisfy the stability condition $s > R - G$, so even though the UAL is never fully paid off, the funded ratio asymptotically approaches its SS value (see Appendix):

$$(10)\qquad f^* = 1 - \left(1 - f°\right)(R/G)^N.$$

This is a very simple solution which can be readily calibrated. Consider the mean actuarial assumptions reported above, $R = 1.077$ and $G = 1.037$, and $N = 30$.[19] Again, we see that if the amortization target $f° = 1$, the SS will be full funding, but for lower amortization targets f^* drops precipitously. If the target funded ratio were set at 80%, the actual SS would be quite a bit lower: 37.8%. Therefore, to achieve a SS of 80%, the target ratio for amortization purposes would have to be much closer to one: 93.6%. To achieve a SS of 70%, a target that is now deemed satisfactory by some,[20] $f°$ would need to be set at 90.4%. To take the extreme case, the minimum target to avoid SS insolvency is $f°_{\min} = [1 - (G/R)^N] = 67.9\%$. That is, a pay-go system can be mimicked with an actuarial funding formula, but the amortization target would have to be set at 67.9%.

Table 1 provides a more expansive picture of the SS funded ratios as a function of the amortization target and (R/G). The bold row represents (R/G) under the mean actuarial assumptions above, $1.077/1.037 = 1.039$. At this value of (R/G), every percentage point reduction in the amortization target reduces the SS funded ratio by over 3 points. That is the slope in Figure 1. Clearly, the SS funded ratio is quite sensitive to variation in the amortization target. Conversely, for each percentage point reduction in the true SS target, one can only cut the amortization target by one-third of a point.

17. "Level dollar" amortization is the special case with $G = 1$.

18. This assumes that the amortization series starting in period t is set to amortize the UAL of period $t - 1$, with interest accrued in period t, i.e., the present value of the amortization series starting in period t is $R \cdot \mathrm{UAL}_{t-1}$. The derivation is a simple application of the sum of a geometric series.

19. We need not consider any of the other actuarial assumptions, such as the inflation rate or life expectancy, since their effects are impounded here in G, R, and/or c^n (which does not enter the solution anyway).

20. Dean Baker, codirector of the Center for Economic Policy Research, as reported in *Politico* and unnamed "investment analysts" cited in the *Baltimore Sun* (Campbell 2014).

TABLE 1
Steady-State Funded Ratios, f*, under *x*% Funding Targets (Equation (10); $N = 30$)

		Amortization target, f°						
		0.70	**0.75**	**0.80**	**0.85**	**0.90**	**0.95**	**1.00**
R/G	**1.02**	0.46	0.55	0.64	0.73	0.82	0.91	1.00
	1.03	0.27	0.39	0.51	0.64	0.76	0.88	1.00
	1.039	**0.07**	**0.22**	**0.38**	**0.53**	**0.69**	**0.84**	**1.00**
	1.05			0.14	0.35	0.57	0.78	1.00
	1.06				0.14	0.43	0.71	1.00
	1.07					0.24	0.62	1.00
	1.08						0.50	1.00

We can now calibrate the degree of generational inequity and the same result holds: it is very sensitive to the target funded ratio. The SS contribution rate, given by Equation (5), can be related directly to the target funded ratio, for given R and G:

$$(11)\qquad c^* = c^n + (c^p - c^n)\left(1 - f^*\right) = c^n + (c^p - c^n)\left(1 - f^\circ\right)(R/G)^N .$$

Just as the SS funded ratio varies by over 3 points for every point in the target ratio (for $R/G = 1.039$), so too for the SS unfunded ratio, $(1 - f^*)$, and, hence, the share of the extra burden to fund the benefits of prior cohorts. For $f^\circ = 80\%$, the share of the extra burden rises not to 20%, but to 62.2%. If 20% generational inequity is acceptable, then the amortization target can only be cut to 93.6%.

The model here suggests how one might think of plans' contribution behavior "as if" a target funded ratio were embedded in the amortization formula, with the ensuing actual funded ratio. To illustrate, consider the national average, that employer contributions covered only 85.3% of the annual required contribution (ARC) in 2013.[21] The ARC rate for these plans averaged 17.6%, of which 8.0% was the employer normal cost and 9.6% amortization. From these data one can infer that the average plan paid 73.3% of the amortization. The same dataset gives $f = 74.1\%$. This is "as if" the target funded ratio $f^\circ = 93.1\%$[22] which implies $f^* = 78.4\%$, by Equation (10). Thus (perhaps coincidentally), these data are consistent with the idea that the average plan is acting as if it were content with an 80% funding target and implicitly uses the corresponding amortization target to pare back contributions. Of course, this average masks wide variation between those plans aiming at full funding and those which further reduce contributions, consistent with a much lower SS.

21. This is for those plans in Boston College's Public Plans database whose members are in Social Security (for their ARCs to be comparable with each other). The figures given are all weighted averages for about 100 plans.

22. The 73.3% figure is $(0.853 \times 0.176 - 0.080)/0.096$. The ratio of amortization "as if" the target ratio were f° to the full-funding amortization is $(f^\circ - f_{t-1})/(1 - f_{t-1})$. Setting this to 0.733, with $f_{t-1} = 0.741$ implies $f^\circ = 0.931$.

VI. ASSUMED RETURN > *R*

The previous analysis helps understand the properties of a system that either builds underfunding into its amortization formula or acts "as if" it has done so by shortchanging the contributions required to reach full funding, with the rationale that "80% is good enough." In many plans, however, the main cause of underfunding is that the fund's assumed return has exceeded the market return in recent years. If that gap is temporary and the fund's performance is expected to revert to longer-term historical norms on which the assumed return is purportedly based, then the "80%" rationale is simply a statement that, while short today, we are still on track to full funding. We may, however, now have an environment where prospective returns are lower than previously assumed. If the assumed rate is maintained or only minimally reduced, then we have a situation analogous to that modeled above, where underfunding is built into the system, through the back door of the assumed return, and is effectively rationalized with the 80% standard. In this section, I formally examine a system where the assumed return exceeds the true return, and provide new insights into the resulting SS.

We begin with some additions to the preceding notation:

$R' = 1 +$ assumed return on investment $> R = 1 +$ true return

$c^{n\prime} =$ measured normal cost (valued at R') $< c^n =$ true normal cost (valued at R)

$L'_t =$ measured liabilities (valued at R') $< L_t =$ true liabilities (valued at R)

$f'_t =$ measured funded ratio at end of period t, $A_t/L'_t > f_t =$ true funded ratio

The dynamic of assets is unchanged from Equation (1), since their evolution rests on actual returns, R, along with actual contributions and benefit payments, c_t and c^p. Equations (2) and (3) continue to represent the dynamic of true liabilities, but for measured liabilities, these analogous expressions hold:

(12) $$L'_t = GL'_{t-1} = R'L'_{t-1} + \left(c^{n\prime} - c^p\right) W_t$$

(13) $$L'_t = \left[G/\left(R' - G\right)\right]\left(c^p - c^{n\prime}\right) W_t.$$

The contribution rate is modified from Equation (6), since R' affects $c^{n\prime}$, L'_{t-1}, and amortization speed s':

(14)
$$c_t = c^{n\prime} + s'\left(L'_{t-1} - A_{t-1}\right)/W_t$$
$$= c^{n\prime} + s'\left(1 - f'_{t-1}\right)\left(c^p - c^{n\prime}\right)/\left(R' - G\right).$$

Here the target is full funding, and we have used Equation (13). The immediate impact on contributions of assuming R′ instead of R is complex, but is presumably negative, due to reductions in the measured normal cost and UAL; it will be instructive to contrast this with the SS impact, below.

A. *Steady State*

Consider first the SS funded ratio. It would not be surprising to find that the true funded ratio is less than 1, since it is below the measured ratio, which is targeted for full funding. It may, however, be surprising that not even the measured ratio—inflated by the higher discount rate for liabilities—reaches 1 in SS. As shown in the Appendix:

(15) $$f'^{*} = \left[s' + G - R'\right]/\left[s' + G - R\right] < 1 \text{ for } R' > R.$$

The reason $f'^{*} < 1$ may be different from what intuition suggests. One might think it has something to do with the low-balling of normal cost. But Equation (15) shows f'^{*} is independent of the relationship between measured and true normal cost. To see why, suppose $f'_{t-1} = 1$ and consider why f'_t falls below 1. Since we are at full funding (as measured using R', i.e., liabilities are low-balled, but assets match them), amortization payments cease and $c_t = c^{n\prime}$ (by Equation (14)). Since $c^{n\prime}$ low-balls true normal cost, the *true* liabilities that accrue are not fully funded. But that is not the problem: *measured* liabilities only accrue at $c^{n\prime}$. Since contributions include normal cost as measured, they cover accruals as measured, so the measurement of normal cost is not the reason the measured funded ratio falls below 1.[23] The problem is that liabilities are rolled forward at R', while assets (net of contributions) only accumulate at rate R.

TABLE 2
Steady-State Measured Funded Ratios, f'^{*}, under High Assumed Returns (Equation (17); $N = 30$)

		Assumed/true rate of return, R'/R						
		1.000	**1.005**	**1.010**	**1.015**	**1.020**	**1.025**	**1.030**
R'/G	**1.02**	1.00	0.83	0.71	0.62	0.55	0.50	0.45
	1.03	1.00	0.80	0.67	0.58	0.51	0.46	0.41
	1.04	1.00	0.78	0.63	0.54	0.47	0.41	0.37
	1.05	1.00	0.74	0.59	0.49	0.42	0.37	0.33
	1.06	1.00	0.71	0.55	0.45	0.38	0.33	0.29
	1.07	1.00	0.67	0.50	0.40	0.34	0.29	0.25
	1.08	1.00	0.62	0.45	0.36	0.29	0.25	0.22

Specifically, consider "level percent" amortization, Equation (9), with assumed return R':

(16) $$s' = \left(R' - G\right)/\left[1 - \left(G/R'\right)^N\right].$$

Substituting in Equation (15), we have:

(17)
$$f'^{*} = \left(R' - G\right)/\left\{\left(R' - G\right) + \left(R' - R\right)\left[\left(R'/G\right)^N - 1\right]\right\} < 1 \text{ for } R' > R > G.$$

Note that as N goes from 1 to infinity, f'^{*} goes from $G/[G + (R' - R)]$ to zero.

To calibrate the system, consider first the benchmark case discussed above: $R' = 1.077$, $G = 1.037$, $N = 30$. If we suppose that the true return is 0.5 percentage points lower, $R = 1.072$, the SS value of the measured funded ratio, $f'^{*} = 79.1\%$. Thus, if one accepts the 80% standard as "good enough," one might be willing to maintain an assumed rate of return that is half a percentage point too high. However, the SS is very sensitive to the assumed return. If the true return is a full point lower, $f'^{*} = 65.4\%$ and if it is two points lower (i.e., 5.7% true return), f'^{*} falls below 50%.

Table 2 illustrates the sensitivity, by depicting f'^{*} for various values of R'/G and R'/R. The values of R'/R vary from 1.000 to 1.030, with increments of 0.005, which correspond approximately to 0.5% increments on the spread between R' and R. Looking across any given row[24] one can see the sensitivity of f'^{*} to the assumed return. The relationship is not linear (f'^{*} only approaches zero asymptotically), but f'^{*} rapidly falls below 50%.

23. Formally, as seen in Appendix Equation (A5), when $c_t = c^{n\prime}$ the terms in $c^{n\prime}$ drop out, no matter how normal cost is measured—even if it were measured accurately.

24. The benchmark case of $R' = 1.077$ and $G = 1.037$ corresponds approximately to the row with $R'/G = 1.04$.

As we have seen in Equation (5), the SS unfunded ratio f^* corresponds directly to the degree of generational inequity, but for our *measured* magnitudes with $R' > R$, there is a twist. Write the SS contribution rate by substituting Equation (16) into Equation (14), evaluated at f'^*:

(18)
$$c^* = c^{n\prime} + \left(c^p - c^{n\prime}\right)\left(1 - f'^*\right) / \left[1 - \left(G/R'\right)^N\right].$$

The potential extra burden of funding prior cohorts is measured as $(c^p - c^{n\prime})$ and the share of that actually borne is $(1 - f'^*)/[1 - (G/R')^N]$. That is, unlike the true burden (given in Equation (5)), each cohort carries a share of the measured burden that *exceeds* the measured unfunded ratio. For example, in our benchmark case ($R' = 1.077$, $G = 1.037$), if R' exceeds R by half a point—so the SS measured funded ratio is about 80%—each cohort carries not 20%, but about 30% of the (measured) extra burden. At a full point spread, that rises to about 50%, and at a two point spread that goes to about 75%.

Finally, it is worth pointing out that even though the initial impact of a policy of high assumed returns is undoubtedly to reduce contributions (both normal cost and amortization), in SS, it raises contributions. We can see this in Equation (5), which still holds here. With $R' = R, f^* = 1$ and $c^* = c^n$, but with $R' > R, f^* < 1$ and $c^* > c^n$.[25] This is no surprise—it is certainly understood that a policy of high assumed returns is a policy of "pay later," i.e., generational inequity. Our contribution here is to shed further light on the mechanism by which this occurs, and to show the order of magnitudes involved. The mechanism is the SS amortization payments generated by the failure of the system to sustain full funding under the *inflated* ratio (let alone the true ratio). That failure, in turn, owes to the inflated growth of measured liabilities from the passage of time (rather than the deflated measure of normal cost, i.e., the accrual of new benefits). The magnitudes of the generational inequity generated by small deviations of assumed from true returns (0.5–2.0 percentage points) may be surprising: 30%–75% of the measured extra burden of funding prior cohorts.

25. Since Equations (5) and (18) both hold in steady-state for c^*, together they define the complex relationship between the measured and true unfunded ratios, $(1 - f'^*)$ and $(1 - f^*)$. Equivalently, we can use Equations (3) and (13) to write $f_t/f'_t = L'_t/L_t = [(R - G)/(R' - G)][(c^p - c^{n\prime})/(c^p - c^n)]$. Although the steady-state value of the measured funded ratio does not depend on the gaps among c^p, c^n, and $c^{n\prime}$, these do matter for the true funded ratio.

VII. CONCLUSION

The "80% standard"—mythical though its origins may be—raises the possibility that steady states may be sustainable with less than full funding. This is true. Indeed, SS funding ratios as low as zero may be sustainable, in a mathematical sense, if the funding formula includes an amortization payment that adjusts outside of SS (to provide stability) and generates sufficient contributions in SS to help fund the benefits of prior cohorts as well as the current cohort. I have shown here that the extra contributions for prior cohorts—the generational inequity—take a simple form in SS: *it is the SS unfunded ratio times the difference between the pay-go rate and the normal cost.*

I have also examined the determinants of the SS unfunded ratio under an "*x*%" amortization formula (either formal or informal) and a policy of high assumed returns. Under an *x*% amortization formula, the SS unfunded ratio (and, hence, the degree of generational inequity), will exceed the amortization target unfunded ratio. Thus, to achieve a 20% SS unfunded ratio will require a target funded ratio for amortization of more than 80%; I calibrate it at about 94% under mean actuarial assumptions. For a 30% SS unfunded ratio—corresponding to the 70% funded target bandied about by some analysts—the amortization target would still need to be about 90% under these assumptions. Even the extreme case of zero funding (pay-go) would require an amortization target of nearly 70% to generate the amortization payments necessary to fund current benefits. This would impose upon each cohort the full extra burden of funding prior cohorts, mimicking the pay-go system that actuarial funding was originally designed to replace.

Under a policy of high assumed returns, with a target funded ratio of 100%, the SS funded ratio—*even as inflated by the high discount rate*—will fall short. I calibrate, under mean actuarial assumptions, that if the goal is a 20% SS unfunded ratio as measured, the assumed return can run half a percentage point above the true return. However, the degree of generational inequity (as measured) will not be 20%, but 30%. If the spread between assumed and actual returns reaches 2 percentage points, the extra burden borne by each cohort can easily reach 75%.

More generally, the main *policy* takeaway of my analysis is that the SS degree of generational inequity is highly sensitive to the policies considered. The "80%" mantra lends a spurious complacency regarding departures from sound finance: small deviations from the 100% funding target or of the assumed return from true returns generate large degrees of generational inequity. To reduce generational inequity it will be necessary to hew closely to the 100% funding target and to significantly mark down assumed returns.

APPENDIX

The dynamic of the funded ratio, $f_t = A_t/L_t$, is found by using Equations (1)–(3):

$$\text{(A1)}\quad f_t = A_t/L_t = RA_{t-1}/GL_{t-1} + \left(c_t - c^p\right) W_t/L_t$$
$$= (R/G)f_{t-1} + \left[(R-G)/G\right]\left(c_t - c^p\right)/\left(c^p - c^n\right).$$

The system evolves from the initial condition f_0 based on the sequence of contribution rates c_t. If c_t is fixed at c, Equation (A1) is a one-variable linear difference equation. The SS funded ratio is found by setting $f_t = f_{t-1} = f^*$, yielding $f^* = (c - c^p)/(c^n - c^p)$. The solution is $f_t = (R/G)^t(f_0 - f^*) + f^*$, and the stability condition is $R < G$.

TARGET FUNDED RATIO < 100%

Endogenizing contributions, under the funding formula (6), we can use Equation (3) to write:

$$\text{(A2)}\quad c_t = c^n + s\left(f^\circ - f_{t-1}\right)\left(L_{t-1}/GW_{t-1}\right)$$
$$= c^n + s\left(f^\circ - f_{t-1}\right)\left(c^p - c^n\right)/(R-G).$$

Substituting Equation (A2) into Equation (A1) and simplifying, we have

(A3)
$$f_t = (R/G)f_{t-1} + \left[(R-G)/G\right]\left[s\left(f^\circ - f_{t-1}\right)/(R-G) - 1\right]$$
$$= \left[(R-s)/G\right]f_{t-1} + \left[sf^\circ - (R-G)\right]/G.$$

Thus, we have again characterized the system with a simple linear difference equation. The stability condition is now $(R-s)/G < 1$, or $s > R - G$. Conversely, to prevent oscillation, one would not want to over-adjust by setting $s > R$, which would more than pay off the (target) unfunded liability in one period.[26] Thus, the economically meaningful range is $s \in (R - G, R]$.[27] As stated in the text, the stability condition is independent of the target funded ratio.[28]

The SS funded ratio f^*, given in Equation (7), is found by setting $f_t = f_{t-1} = f^*$ in Equation (A3).

For "level percent" funding, the system dynamic is found by substituting Equation (9) into Equation (A3):

$$\text{(A4)}\quad f_t = \left\{\left[G - R\,(G/R)^N\right]/G\left[1 - (G/R)^N\right]\right\}f_{t-1}$$
$$+ \left[(R-G)\left(f^\circ - 1 + (G/R)^N\right]/G\left[1 - (G/R)^N\right].$$

Solving Equation (A4) for $f_t = f_{t-1} = f^*$ (or substituting Equation (9) into Equation (7)), we have $f^* = 1 - (1 - f^\circ)(R/G)^N$.

ASSUMED RETURN > R

Equation (A1) still holds for the dynamic of the true funded ratio, f_t, but using Equations (1), (12), and (13), we find the following dynamic for the measured funded ratio:

(A5)
$$f'_t = (R/G)f'_{t-1} + \left[\left(R' - G\right)/G\right]\left(c_t - c^p\right)/\left(c^p - c^{n\prime}\right).$$

More specifically, substituting Equation (14) into Equation (A5), we have:

$$\text{(A6)}\qquad f'_t = \left[\left(R - s'\right)/G\right]f'_{t-1} + \left[s' + G - R'\right]/G.$$

The stability condition, $s' > R - G$, is analogous to that of the target funding model.[29] The SS funded ratio f^*, given in Equation (15), is found by setting $f_t = f_{t-1} = f^*$ in Equation (A6). Note from Equation (A6) that if $f'_{t-1} = 1$, f'_t drops to $1 - (R' - R)/G$, and will continue dropping until reaching f^*.

26. Consider the borderline case $s = R$. If the target is full funding, then, as Equation (A3) shows, for any funded ratio f_{t-1}, contributions over the next period bring f_t back to unity. More generally, for $f^\circ < 1$, the steady state, given in Equation (7), will also be attained in one period. For the other borderline case, $s = R - G$, f_t will either remain stationary at any arbitrary level (if $f^\circ = 1$) or will decline indefinitely (if $f^\circ < 1$); as shown in Equation (7), there is no well-defined steady state.

27. The two conditions can be written as: $R(f^\circ L_{t-1} - A_{t-1}) \geq (c_t - c^n)W_t > (R - G)(f^\circ L_{t-1} - A_{t-1})$. The first term is principal plus interest on the (target) UAL, the middle term is the amortization payment (from Equation (6)), and the last term is the growth-adjusted interest on the (target) unfunded liability. One does not need to pay full interest for the funded ratio to be stable, since payroll growth will help erode the unfunded ratio.

28. The behavior of the funded ratio, Equation (A3), is also independent of the relationship between c^p and c^n, unlike the more general formulation given in Equation (A1). That is, the conventional actuarial formula for contributions, slightly generalized in Equation (A2), eliminates the terms in c^p and c^n from the funded ratio dynamic, since $(c_t - c^p)$ in (A1) is now proportional to $(c^p - c^n)$. Specifically, this follows from the fact that normal cost, net of pay-go cost appears on both the liability accrual (2) and asset accrual (1) under contribution formula (A2).

29. Once again, this dynamic does not depend on normal cost (true or measured, c^n or $c^{n\prime}$) or the cost of benefits (c^p). The reason is the same: normal cost—however measured—net of pay-go cost appears in both the liability accrual and asset accrual, so it vanishes from the dynamic of the measured funded ratio.

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