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Second-best subsidies in monopolistic competition

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In the Spence–Dixit–Stiglitz model of monopolistic competition, an output (or input) subsidy is welfare-improving, under competitive conjecture. The significance of this result is that it holds even when the subsidy exacerbates the potential problem of excess product diversity.

1. Introduction

Models of monopolistic competition, such as Dixit and Stiglitz [1977], and Spence [1976], typically exhibit two market failures in the differentiated sector: insufficient resources allocated to the sector (due to pricing above marginal cost); and suboptimal product diversity, as resources are generally spread over too few or too many products.

Fiscal remedies to achieve the first-best optimum require two instruments to address the two market failures: A subsidy on output (or inputs), to restore marginal cost pricing; and a tax or subsidy on product introduction, to achieve optimal product diversity. Although there are difficulties with both instruments, they are probably more severe with the latter, so a second-best approach relies on the output (input) subsidy alone. Such a subsidy serves both to attract more resources and more products to the sector. In the case of excess product diversity, this raises the question of whether such a subsidy should be positive. The present note shows that it should, at least under competitive conjectures. [1]

2. Preferences, technology and equilibrium

Following Dixit and Stiglitz (1977) and Spence (1976) we model the benefits of variety by assuming that all consumers are alike, but have strictly convex preferences over a large number of

[1] Myles (1989) provides a general analysis of optimal product tax rules in general equilibrium models with imperfect competition. However, the number of products is fixed in the model (except for Appendix A.1), and the issue of optimal product diversity is not addressed.

potential differentiated outputs, $\tilde{n}$, of which only n will actually be produced. Preferences take the following separable form:

$$U = U\left[S \equiv \sum_{i=1}^{\tilde{n}} g(y_i),\ L \equiv 1 - X\right], \qquad g(0) = 0, \quad g' > 0. \tag{1}$$

Here, L is the good from the undifferentiated sector (e.g. leisure, or, if labor is supplied inelastically, L can be some homogeneous output produced under constant returns to labor). Units are normalized such that the maximum amount of the undifferentiated good available is unity, and X of it is used in the differentiated sector. The y's are the symmetrically differentiated products, and S is a summary statistic for them. This formulation includes the CES case, where the elasticity of g with respect to y is constant ($g = y^{\epsilon}$, of which the degenerate case $\epsilon = 1$ is homogeneous products), but we will be more interested in the variable elasticity case:

$$\epsilon(y) \equiv yg'(y)/g(y).$$

The representative consumer maximizes (1) subject to:

$$\sum_{i=1}^{n} p_i y_i = X + \sum_{i=1}^{n} \pi_i, \tag{2}$$

where π_i is the profit from product i, and L is numéraire. [2] First-order conditions for an interior optimum give us the price of good i:

$$p_i = (U_S/U_L)g_i' \equiv M(S,\ X)g'(y_i), \qquad M_S,\ M_X < 0, \tag{3}$$

where M is the marginal rate of substitution between sectors (U_s/U_L), and subscripts (other than i) denote partial derivatives. [3] To satisfy second-order conditions, and to rule out boundary solutions, we require

$$g''(y) < 0 \quad \text{and} \quad \epsilon(y) \equiv yg'(y)/g(y) < 1 \tag{4}$$

in the vicinity of symmetric equilibrium (to which we will confirm our attention). [4] Production of the differentiated products,

$$y_i = f(x_i), \quad i = 1, \ldots, \tilde{n}, \quad f(0) = 0, \quad f' > 0,$$

exhibits a non-convexity, sufficiently pronounced that only $n < \tilde{n}$ goods will be produced in equilibrium. [5]

[2] This is a non-Walrasian consumer, since the summations go to n, rather than $\tilde{n}$. That is, consumers are rationed (to zero) on the $(\tilde{n} - n)$ inactive markets, as pointed out by Hart (1979, 1985a, 1985b) and Jones (1987). Without such rationing, free-entry equilibrium does not exist with a production non-convexity, as emphasized by Costrell (1986).

[3] Spence (1976) and others assume that utility is linear in L, such that X does not appear in (3).

[4] The elasticity condition implies $ng(\Sigma y/n) > (n-1)g(\Sigma y/(n-1))$, so consumers do indeed spread their expenditures in the differentiated sector over all n products offered.

[5] This formulation assumes there are no interdependencies among a firm's product lines, so there is no loss of generality in identifying a firm with a product. Ireland (1983) considers the case where there is an inter-product economy.

The profits from product i can be written in terms of x_i:

$$\pi_i = p_i y_i - x_i = M(S, X) g'(f(x_i)) f(x_i) - x_i. \tag{5}$$

In choosing x_i, the firm must form a conjecture regarding the effect of its choice on $M(S, X)$. We consider the competitive conjecture that $M(S, X)$ is taken as given. [6] This conjecture is rationalized by Chamberlin (1962) by appeal to 'large group' considerations, and has been commonly adopted since Dixit and Stiglitz (1977) and Spence (1976), even when n is not large. An alternative basis for this conjecture has been given by Costrell (1990) [extending a point by Perry (1982)], showing that under free entry it is the consistent conjecture.

Under this conjecture, the slope of the firm's perceived demand curve for any product is Mg''. The profit-maximization condition is

$$M(S, X)(g' + yg'')f' = 1, \tag{6}$$

where subscripts have been dropped, and the free-entry condition is

$$M(S, X) g' y = x. \tag{7}$$

By (6), price (Mg') exceeds marginal cost ($1/f'$), since the demand curve slopes down ($Mg'' < 0$). By (6)–(7), the output elasticity

$$xf'/y = g'/(g' + yg'') > 1, \tag{8}$$

so production occurs in the region of increasing returns, the usual result from monopolistic competition.

3. Optimality and market failure

The social welfare function can be written as a function of X and n:

$$W(X, n) \equiv U(ng(f(X/n)), 1 - X), \quad \text{where} \quad X = nx. \tag{9}$$

The first partials, expressed in terms of the numéraire L, are

$$W_X/U_L = M(S, X) g' f' - 1, \tag{10}$$

$$W_n/U_L = M(S, X)(g - xf'g'). \tag{11}$$

The first-best optimality condition for X, setting (10) to zero, is simply marginal cost pricing. The optimum for n, from (11), is also in the region of increasing returns [$xf'/y = 1/\epsilon(y) > 1$,, by (4)]. That is, as Chamberlin and others since have pointed out, to achieve optimal product diversity, resources in the differentiated sector should be spread over so many firms that some economies of scale are sacrificed. Of course, the free market need not pick the optimal point in that region.

At the free market configuration (6)–(7), we find

$$W_X/(U_L \cdot M(S, X)) = W_X/U_S = -yg''f' > 0, \tag{10'}$$

$$W_n/(U_L \cdot M(S, X)) = W_n/U_s = \left[(g' + yg'')g - (g')^2 y\right]/(g' + yg''). \tag{11'}$$

[6] However, see footnote 10.

As (10′) shows, there are insufficient resources allocated to the differentiated sector, due to the failure of marginal cost pricing. The fiscal remedy is a positive subsidy on either output or input. [7]

From (11′), the sign of the bracketed term determines whether there is insufficient or excess product diversity, given the resources allocated to the sector. It is readily verified that this term has the same sign as $\epsilon'(y)$. In the borderline CES case, these resources are spread over the optimal number of products. [8] In the case of $\epsilon'(y) > 0$ [as in the example of Dixit and Stiglitz (1977, Section II)], there is insufficient product diversity, and for $\epsilon'(y) < 0$ [as in Pettengill (1979)], there is excess product diversity. In these cases, the fiscal remedy would be a lump-sum subsidy or tax on product introduction. [9] Although such policies are observed from time to time in the case of one-product firms (license fees, medallions, etc.), it is difficult to conceive of such policies in the common case of multi-product firms, and would, in fact, be an incentive to create such firms in the case of a tax.

4. The second-best subsidy is positive

Output and input subsidies are perhaps more common, so it may be worth investigating their second-best properties. One would expect such a subsidy to attract more resources to the sector and to increase the number of products offered, which would obviously improve welfare on both counts, if there were insufficient product diversity. In the case of excess product diversity, however, it is not obvious that the subsidy should be positive. That is, it remains to be shown that for subsidy s

$$\mathrm{d}W(X, n)/\mathrm{d}s = W_X\, \mathrm{d}X/\mathrm{d}s + W_n\, \mathrm{d}n/\mathrm{d}s > 0 \tag{12}$$

when $s = 0$, so a positive subsidy will improve welfare, even when $W_n < 0$.

Suppose the subsidy is on input (it could just as well be on output), so the RHS of (6)–(7) are multiplied by $(1 - s)$. Then (8) still holds, which determines x independently of s, i.e. $\mathrm{d}x/\mathrm{d}s = 0$. Therefore, $\mathrm{d}X/\mathrm{d}s = \mathrm{x} \cdot \mathrm{d}n/\mathrm{d}s$, and (12) simplifies to

$$\mathrm{d}W/\mathrm{d}s = (xW_X + W_n)\, \mathrm{d}n/\mathrm{d}s.$$

From (10′) and (11′), and using (8), we find that at $s = 0$,

$$\begin{aligned}(xW_X + W_n)/U_S &= -xyg''f' + g - (g')^2 y/(g' + yg''),\\ &= g - yg' > 0,\end{aligned} \tag{13}$$

[7] The optimal subsidy on output is $s_y = -yg''/(g' + yg'')$ (evaluated at the optimum), while the equivalent subsidy on input is $s_x = -Myg''f'$. The significance of these subsidies, of course, is only relative to the undifferentiated sector. For example, the differentiated sector's subsidy could be financed out of a tax on the undifferentiated sector.

[8] Dixit and Stiqlitz (1977, Section I) argue there is insufficient product diversity in this case, since $n < n^*$ (where * denotes optimal values). However, this reflects the paucity of resources in the sector, due to the market power problem, rather than the product diversity problem *per se*. That is, the nature of the suboptimalities are more apparent from the signs of W_n and W_x than from the signs of $(n^* - n)$ and $(X^* - X)$. For example, consider the case of homogenous products ($\epsilon = 1$). Then with U-shaped average cost, free entry, and non-competitive conjectures (e.g. Cournot–Nash oligopolists), we find $W_x > 0$ and $W_n < 0$: there are too few resources and they are spread over too many firms. However, it is likely that $n < n^*$. If so, this clearly reflects the resource restriction, in the exercise of market power. This is the proper interpretation of the Dixit–Stiglitz CES result as well.

[9] The value of such a subsidy or tax is given by the RHS of (11′), multiplied by $M(X, X)$ and evaluated at the optimum.

by (4). Therefore, $\mathrm{d}W/\mathrm{d}s > 0$ if only $\mathrm{d}n/\mathrm{d}s > 0$. It is readily verified that this is the case, using the zero profit condition: [10]

$$\mathrm{d}n/\mathrm{d}s = -x/(M_S g + M_X x)\, y g' > 0, \tag{14}$$

by (3).

5. Conclusion

This note has shown that under monopolistic competition, an output or input subsidy to the differentiated sector is welfare-improving, even if there is excess product diversity. This result follows almost immediately from the invariance of output per product with respect to the subsidy, under competitive (or, slightly more generally, constant-elasticity) conjectures. For if output per product is invariant, then zero profits are only restored by an increase in the number of products. That increase does not spread given resources over more products, which would exacerbate the problem of excess product diversity. Rather, it is simply the vehicle by which more resources are brought to the sector, ameliorating the market power problem.

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[10] For non-competitive conjectures, $\mathrm{d}M/\mathrm{d}x_i < 0$, it can readily be shown that the market power problem is more pronounced, strengthening the case for the subsidy. On the other hand, this tilts the product diversity problem towards excess diversity, as shown by Koenker and Perry (1981), which weakens the case for the subsidy.

In the simple case where the conjecture is constant in elasticity form [i.e., $(x_i/M)(\mathrm{M}/\mathrm{d}x_i)$ is constant, but not necessarily zero], the analysis of this section goes through as before, and the optimal subsidy is still positive. This slight generalization, however, does not cover the Cournot–Nash case, for which analytical results could not be obtained. A series of simulations turned up no counter-examples, though.