A Simple Model of Educational Standards

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I model standards for educational credentials, such as high-school diplomas. Standard-setters maximize their conception of social welfare, knowing that utility-maximizing students choose whether to meet the standard. I show that more egalitarian policymakers set lower standards, the median voter would prefer higher standards (under symmetric distributions), and decentralization lowers standards (among identical communities). Optimal standards do not necessarily fall with increased student preference for leisure, deterioration of nonstudent inputs to education, or increased student heterogeneity. Superseding binary credentials by perfect information increases average achievement and social welfare, for plausible degrees of heterogeneity, egalitarianism, and pooling under decentralization. (JEL 121)

"Everybody has won, and all must have prizes."
—The Dodo, in Alice's Adventures in Wonderland, by Lewis Carroll

Student time and effort are arguably the most important inputs to education, for given levels of ability. These student inputs respond to educational standards, either rising to meet high standards or dropping off in discouragement. But what determines standards?

The determination of standards has been the subject of casual speculation, but little formal modeling.¹ Some common views in the public debate are:

(i) The educational establishment sets low standards for egalitarian reasons (Diane Ravitch, 1985; Rita Kramer, 1991).

(ii) In a related view, the lack of accountability allows schools to set standards below what parents would like (Chester E. Finn, 1991).

(iii) The U.S. system of decentralization, with no national curriculum or exams, results in lower standards than abroad (Finn, 1991).

(iv) The emphasis of youth culture on current leisure and consumption leads schools to acquiesce in low standards (Theodore R. Sizer, 1984).

(v) Deterioration of nonstudent inputs, such as teacher quality (W. Timothy Weaver, 1983) and parental inputs (James S. Coleman, 1991), makes it more difficult for schools to maintain high standards.

(vi) Increased “diversity” of the student population leads schools to lower standards, compared to more homoge-

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Andrew Weiss (1983) has an educational standard for a binary credential, but it is exogenous. Joseph E. Stiglitz (1975) endogenizes educational “intensity,” but students exercise no choice with respect to effort. Weiss and Stiglitz focus on screening and sorting issues; this paper is a simple human-capital model.

The empirical significance of standards has also been little studied. Jonathan E. Jacobson (1992) finds evidence in the National Longitudinal Survey of Youth that in states where minimum-competency graduation tests were introduced, the performance of bottom-quintile students improved, while performance in the middle three quintiles deteriorated. Charles Murray and Richard Herrnstein’s (1992) analysis of other data paints a consistent picture. The positive results have some relevance to the present model, while the negative results are concentrated among the college-bound, who are excluded here.
neous countries (Albert Shanker [1992] cites this view to debunk it).
(vii) Depressed earnings of dropouts, due to skill-biased technical progress and factor-price equalization, make high standards costly.

Although all of these propositions may sound obvious to their advocates, formal analysis can distinguish those which are robust from those which are not and, more importantly, can indicate why.

I propose a simple model of educational standards with three features: (a) the standard is defined as the required level of proficiency for a binary credential, such as the high-school diploma; (b) the standard is set by policymakers who maximize their conception of social welfare; and (c) utility-maximizing students choose whether to meet the standard.

The first assumption, that only a binary credential is relevant, rests on the finding developed in some detail by John H. Bishop (1988, 1990) that employers of high-school graduates rely almost exclusively on the diploma, rather than the fuller information contained in transcripts or employment tests. Schools have little incentive to provide transcripts when asked, and Bishop (1991) points out that fear of Equal Employment Opportunity Commission (EEOC) action has deterred many firms from requesting transcripts. In Section VI, the binary credential model will be compared with one of full information flows, which is the goal of Bishop’s policy recommendations.

The second assumption, that standard-setters maximize their own social welfare function, is best thought of as reflecting their bureaucratic independence from citizen-parents, in a world where voter turnout is low and information is sparse in school committee elections. Public-opinion polls, for example, typically show that by wide margins parents believe standards are set too low. This is consistent with voluminous evidence that other popular measures, like teacher standards, alternative certification, merit pay, and so forth are frequently blocked by National Education Association affiliates and other educational lobbies, or simply ignored and undermined once they are enacted (Thomas Toch, 1991). The model will be compared, in Section III, with a median-voter model, in which parents vote individualistically, to maximize the income of their own children.

The third assumption, that students maximize utility, is routine. The question arises, however, whether standard-setters share those preferences, or whether they assign less value to student leisure. The latter is surely suggested by mandatory schooling laws and also by evidence that students hold an unduly high discount rate (e.g., George F. Loewenstein and Richard H. Thaler, 1989). The basic model below makes the simplifying assumption that standard-setters assign no value to student leisure, an assumption which is relaxed in Section II.

Under these assumptions, policymakers set the standard to elicit their preferred behavior from reluctant students, as in the principal–agent problem. The marginal benefit of a higher standard is the increased productivity of those who continue to meet it, and the marginal cost is the reduced productivity of those who cease to do so. Some of the main implications of the model for the popular propositions (i)–(vii) given above, can be summarized.

2 Costrell (1993a) considers a more complex model, with three standards: high-school graduation, college admission, and college graduation.
3 Byron W. Brown and Daniel H. Saks (1975) introduce an educational policymaker’s objective function, stressing distributional considerations in a mean-variance formulation. However, they do not explicitly model educational standards.

4 The Business Week /Harris Poll reports that 80 percent favor “raising requirements for passing courses and graduating” (14 September 1992).
5 Gary S. Becker (1983) argues that political pressure groups with focused interests are more effective than the general voter, with diffuse interests. Sam Peltzman (1992) models and tries to estimate the effect of teachers’ unions on public education. John E. Chubb and Terry M. Moe (1990) also stress the bureaucratic nature of decision-making in public education.
(i) Egalitarian policymakers do indeed set lower standards. This robust result holds not only for standard-setters concerned solely with students' future incomes; it holds also for standard-setters who value student leisure, even if they value it more than the vast majority of students value it for themselves. What if policymakers try, instead, to respect consumer sovereignty, by aggregating student preferences? Then the effect of egalitarianism will depend on whom the policymakers consider worse off: students of equal income who consider themselves overworked or underworked. This comparison ultimately rests on policymaker preferences, not student preferences.

(ii) The median voter parent, maximizing the child’s future income, will prefer a higher standard than that set by any autonomous policymakers, egalitarian or not, provided the distribution of students with respect to the maximum standard they will meet is symmetric unimodal.

(iii) Decentralization lowers standards among identical communities, an unsurprising result, given that the benefits of high standards are not fully appropriated by the district. An additional reason for this result is on the cost side: high standards deter more students under decentralization.

(iv)-(vi) Perhaps surprisingly, neither a shift of student preferences toward leisure, nor a deterioration of nonstudent inputs, nor increased “diversity” (interpreted here as student variance) necessarily leads policymakers to acquiesce in lower standards. Depending on the degree of egalitarianism, such shifts may lower the marginal cost of higher standards by reducing the number of students on the graduation margin, leading the policymaker to raise standards.

(vii) Although skill-biased technical progress increases the cost of nongraduation, it also improves student incentives to meet the standard, so the optimal standard may not drop.

In Section VI, I consider Bishop’s recommendation of improving the information flow to employers, beyond that of a binary credential. For a near-homogeneous population, this will reduce student achievement, since pass/fail standard-setters can maneuver students into less leisure than they would like. For more likely degrees of heterogeneity, however, perfect information raises student achievement and social welfare, especially compared to that obtained by egalitarian policymakers or those operating in a decentralized setting. Concluding remarks comment briefly on some of the normative implications.

I. The Basic Model

A. Student Behavior

Consider student i with preferences $U'(L_i, w_i)$, defined over leisure $L_i$ and future earnings, $w_i$. The student’s future productivity in the workplace (marginal and average) is governed by the cognitive and social skills acquired in school, by dint of hard work. Thus, student i’s educational production function is $y'(L_i, y^m, y^d < 0$, which gives future productivity as a concave, decreasing function of student leisure. Let $L_0$ be the maximum amount of leisure, and $y_0 = y'(L_0)$ be the zero-effort level of productivity, which is assumed to be the same for all students. It will be notationally convenient to use the inverse production function, $L_i = f'(y_i)$. (A summary of the notation used in this paper is provided in Table 1.)

If there were perfect information, then employers could distinguish individual productivity and pay corresponding wages. Each student’s production function, $y'(L_i)$, would form the constraint in $L_i – w_i$ space, and he or she could readily optimize, as in Figure 1. Students could each pick different levels of effort and earn corresponding wages. In such a world (discussed further in Section VI), educational standards, as modeled in this paper, play no role. As discussed above, Bishop (1988, 1990) has shown that such conditions do not currently obtain. For vari-

6According to Harold W. Stevenson and James W. Stigler (1992), Asian views tend to focus on variation in “ability.” This corresponds to letting $U'(L_i, w_i)$ vary by i (e.g., varying discount rate on future income), but not $y(L_i).
The discontinuity is consistent with the well-documented diploma effect in earnings functions (e.g., Weiss, 1988). With a stochastic element to student success at meeting the standard, the sharp corners of the constraint would be smoothed out, resulting in a reverse-S-shaped relationship between expected future wage and student leisure. Still, the student’s optimal effort level rises and then drops off discontinuously as the standard is raised (Suk Kang, 1985), much like the present nonstochastic model.

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students sort themselves into two groups: one of size $F(\hat{y})$, exerting zero effort, and earning $y_0$; and one of size $[1 - F(\hat{y})]$, exerting the effort required to meet the standard, and earning $\hat{y}$. As the standard is raised over $(y_0, y_{\text{max}})$, more students will opt out of graduating, $F'(\hat{y}) = f(\hat{y}) > 0$, until no students graduate for $\hat{y} > y_{\text{max}}$.

B. The Optimal Standard

Suppose the standard is set by a centralized policymaker, as in some cases abroad, and as may come with national standards and testing under development in the United States (decentralization is treated in Section IV). Assume also that the policymaker is only concerned with students' future incomes, not leisure (considered in Section II). The policymaker is disinterested, in that the objective function is symmetric across students. The policymaker is best viewed here as autonomous, insulated from the electorate (voting is discussed in Section III).

Let the objective function (or a monotone transformation of it) be additively separable, with each student's future income evaluated by a concave increasing function, $h$. If it is strictly concave, the policymaker is egalitarian to some degree; if it is linear then he or she simply maximizes the sum of incomes. In reduced form, the objective function is

$$V(y) = \left[1 - F(\hat{y})\right]h(\hat{y}) + F(\hat{y})h(y_0).$$

Under stated assumptions, there is an interior maximum on $(y_0, y_{\text{max}})$. The first-order condition for the policymaker's optimal standard, $\hat{y}^*$, is:

$$V'(\hat{y}^*) = \left[1 - F(\hat{y}^*)\right]h'(\hat{y}^*) - f(\hat{y}^*)\left[h(\hat{y}^*) - h(y_0)\right] = 0.$$

The first term represents the marginal social benefit of a rise in the standard: the $1 - F$ students who continue to meet it will exert greater effort and become more productive. The second term represents the marginal social cost of a rise in the standard: $f$ students on the margin of graduating will cease to do so and will drop to a lower level of productivity ($y_0$ vs. $\hat{y}^*$).

As might be expected, egalitarian standard-setters choose lower standards than income-maximizing ones, since egalitarians place greater weight on the income losses of those who are discouraged by a high standard than on the gains of those who meet it. Formally, let the income-maximizing solution to (2) be $\hat{y}^+$:

$$(2') \quad \left[1 - F(\hat{y}^+)\right] = f(\hat{y}^+)(\hat{y}^+ - y_0).$$

Thus, at $\hat{y}^+$, an egalitarian policymaker evaluates (2) to find

$$\text{sgn}V'(\hat{y}^+) = \text{sgn}(\hat{y}^+ - y_0)h'(\hat{y}^+) - [h(\hat{y}^+) - h(y_0)] < 0$$

by strict concavity. As egalitarian policymakers lower standards to raise the graduation rate, they reduce GDP, providing an important example of the classic trade-off between the size of the pie and its equal division.

II. Standards with Positive Valuation of Student Leisure

A. Policymaker Preferences Over Student Leisure

Suppose the standard-setter values student leisure, in addition to income. In this respect, the basic model is generalized, but I restrict myself here to the case in which all students share the same inverse production function $\ell(y)$. Then it can be shown that egalitarianism still lowers standards, even if the standard-setter values leisure more than the vast majority of students. The reason is that the standard will be set such that the marginal student, indifferent between graduating and not, values leisure more than the standard-setter. Thus, even though the standard-setter may place a great value on leisure, he or she still views the marginal student as better off graduating than not. A
more egalitarian standard-setter will be more concerned with that student and will lower standards to get that student to graduate.

To see this, let Figure 1 now represent the standard-setter's utility function of student leisure and income, call it $U^*(L, w)$. This assumes that the standard-setter values leisure enough that a tangency exists, but less so than the most leisure-loving students, who prefer $(L_0, y_0)$. Given the inverse production function $f(y)$, utility can be expressed as a composite function of income alone, call it $U^*(y) = U^*(f(y), y)$. Obviously, as we move up the production function, from $(L_0, y_0)$ to the tangency, $U^*(y)$ increases to a maximum and then decreases. It is concave in the vicinity of the tangency, and I make the stronger assumption that it is concave throughout the relevant region.

Aggregating over students, the objective function is now

$$V(\hat{y}) = [1 - F(\hat{y})] h(U^*(\hat{y}))$$

and the first-order condition is

$$V'(\hat{y}^*) = [1 - F(\hat{y}^*)] h'(U^*(\hat{y}^*)) U^*(\hat{y}^*)$$

$$- f(\hat{y}^*) [h(U^*(\hat{y}^*)) - h(U^*(y_0))] = 0.$$ 

A solution $\hat{y}^*$ exists between the tangency and the zero-effort point, $(L_0, y_0)$. The policymaker would prefer all students to be at the tangency, but some students would choose not to graduate rather than work that hard, so the policymaker sets the standard lower, at $\hat{y}^*$, to keep the number of nongraduates down.

The key point here is that the policymaker prefers the outcome of the graduate at $\hat{y}^*$ to that of the nongraduate. The marginal student, by contrast, is indifferent. No matter how much the policymaker values leisure, if there are some students who value it more [steeper indifference curve through $(L_0, y_0)$], the standard will be set such that one of them is the marginal student. This means that any standard-setter, egalitarian or not, would like the marginal student to graduate, but the egalitarian is more inclined to lower standards to that end.

Formally, consider the standard chosen by a nonegalitarian standard-setter, with $h(U^*) = U^*$. Following the same logic as in (2') and (3), an egalitarian finds that at the nonegalitarian optimum, call it $\hat{y}^*$,

$$V'(\hat{y}^*) = [U^*(\hat{y}^*) - U^*(y_0)] h'(U^*(\hat{y}^*))$$

$$- h(U^*(\hat{y}^*)) - h(U^*(y_0))] < 0$$

by strict concavity of $h$. Thus, an egalitarian chooses a lower standard. As in the basic model, the egalitarian places less weight on the utility gains (as measured by the standard-setter, not the student) of the inframarginal students who would rise to meet a higher standard, and more weight on the losses of the marginal students who would be discouraged by it.

B. Aggregating Student Preferences

How does a policymaker set the standard, when he or she uses the students' own preferences to evaluate outcomes, and how does the policymaker's egalitarianism affect that choice? Here, the standard-setter is not concerned about whether the marginal student graduates, because the marginal student is himself indifferent. However, the standard-setter must now weigh the fortunes of inframarginal graduates against one another, since raising standards affects graduates differently, according to their own preferences. The effect of egalitarianism depends on which graduates are considered

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8At the zero-effort point, the second term vanishes, the first term is positive, and $V' > 0$. At the tangency, $U^* = 0$, so the first term vanishes, the second term is negative, and $V' < 0$. Somewhere in between, $V' = 0$.\n
worse off by the standard-setter, and whose interests are thereby deemed more pressing. The problem is that this interpersonal comparison depends on the cardinalization chosen by the standard-setter, a somewhat arbitrary decision. Thus, even though the policymaker is trying to divorce the decision from his or her own views by focusing on student preferences, this turns out to be impossible.

Formally, given some cardinalization, each student's preferences can be represented as a composite function of $y$, $U'(y) = U'(\ell'(y), y)$, where $\ell'(y)$ can again vary by $i$. $U'(y)$ is concave in the vicinity of the student's tangency and is assumed to be concave throughout. Consider a Benthamite standard-setter, who simply aggregates over student preferences. The objective function, then, is

$$V(\hat{y}^B) = \int_{y_0}^{\hat{y}^B} f(\tilde{y}_i) U'(y_0) d\tilde{y}_i + \int_{\hat{y}^B}^{Y_{max}} f(\tilde{y}_i) U'(\hat{y}) d\tilde{y}_i.$$  

The first-order condition for the Benthamite standard, $\hat{y}^B$, is

$$V'(\hat{y}^B) = f(\hat{y}^B) [U''(\hat{y}^B)]_{y_0}^{\hat{y}^B} + \int_{\hat{y}^B}^{Y_{max}} f(\tilde{y}_i) U'(\hat{y}) d\tilde{y}_i.$$  

The first term vanishes, since the marginal student, $i^*$, is indifferent between meeting the standard and not. The second term balances the benefits, for those with $U''(\hat{y}^B) > 0$, and the costs, for those with $U''(\hat{y}^B) < 0$, of raising the standard. The benefits accrue to those who would prefer a higher standard (their tangency is above $\hat{y}^B$—they would work harder if there were a payoff to it); and the costs are incurred by those who meet standard $\hat{y}^B$ but would prefer a lower one (their tangency is below $\hat{y}^B$—they would forgo future income for current leisure).

Now consider how the optimal standard just described for the Benthamite is evaluated by an egalitarian policymaker, with a concave social evaluation of student utility, $h[U'(y)]$:

$$V'(\hat{y}^B) = \int_{\hat{y}^B}^{Y_{max}} f(\tilde{y}_i) h'[U'(\hat{y})] U''(\hat{y}) d\tilde{y}_i.$$  

The sign of this depends on the question of whom the policymaker considers to be better off: the graduates who would prefer a lower standard ($U''(\hat{y}^B) < 0$), or a higher one ($U''(\hat{y}^B) > 0$). Consider three cases.

First, suppose the policymaker values leisure equally across students but judges the graduates who prefer a lower standard ($U''(\hat{y}^B) < 0$) to be more fortunate by virtue of their lesser capacity to enjoy future income (e.g., a higher discount rate). More weight, then, will be given to their concerns (high $h'[U'()]$), so (9) will be negative. Egalitarianism lowers standards, as in the preceding subsection.

Second, consider the converse case, in which the policymaker values income equally across students but judges these graduates with $U''(\hat{y}^B) < 0$ to be more fortunate by virtue of their greater capacity to enjoy leisure. Here, egalitarians give less weight to them and choose higher standards.

Third, suppose the policymaker values income equally, as in the second case, but now considers those graduates with $U''(\hat{y}^B) < 0$ to be less fortunate by virtue of their greater disutility of labor. Egalitarianism now lowers standards again.

These last two cases are identical from the point of view of the students, with quasi-linear utility differing only in the constant term. The egalitarian policymaker,

$$9$$

In the second case, $U'(y, L) = y + g'(L)$, and $g'(0)$ is the same for all $i$; in the third case, $U'(y, L) = y + g'(L) - g'(L_0)$. 

9In the second case, $U'(y, L) = y + g'(L)$, and $g'(0)$ is the same for all $i$; in the third case, $U'(y, L) = y + g'(L) - g'(L_0)$. 


however, makes interpersonal comparisons on the basis of that constant, which is assigned by the policymaker, not the student. In short, the idea of substituting consumer sovereignty of the students for the policymaker’s own preferences can only go so far: it tells the policymaker not to worry about the marginal student’s graduating but cannot tell the policymaker how to weigh the interests of inframarginal graduates against one another. The policymaker must fall back onto a cardinalization which is either arbitrary or which reflects his or her own preferences, as in Subsection II-A.

More importantly, regardless of the cardinalization among graduates, it seems hard to square the deep concern over nongraduates, evinced by policymakers, with the assumption that they respect student sovereignty on this point. For these reasons, I conclude that the preceding subsection’s model carries more force and that egalitarianism lowers standards, regardless of the policymaker’s view of student leisure. For the remainder of this paper, I revert to the special case of Subsection II-A given in the basic model of Section I, in which student leisure is not valued.

### III. The Median Voter’s Optimal Standard

Why do public-opinion polls indicate that most citizens prefer a higher standard than policymakers set? One possible interpretation, suggested by the foregoing analysis, is that the educational establishment is more egalitarian than the median voter (or opinion-poll respondent). Another possibility is that the median voter prefers a higher standard out of individualistic concern for his or her child’s interest, quite aside from his views regarding egalitarianism. In this section I will show that this is indeed the case, if the distribution of students, \( f(\tilde{y}) \), is symmetric unimodal.

Suppose each voter wishes to maximize his or her child’s future income (again, giving no weight to the child’s leisure). The parent is constrained by the child’s behavior, based on the child’s preferences for leisure and income (of which the parent is aware). Each parent, therefore, would like the standard to be set at that level which pushes the child to his or her limit, \( \tilde{y} \).

Assuming all voters have the same number of children, the median voter’s optimal standard is \( \tilde{y}_m \), the maximum standard that will be met by the median student.\(^{10}\)

Consider the median of a symmetric unimodal distribution, \( \tilde{y}_m = (y_0 + y_{\text{max}})/2 \). By the assumption of unimodality, \( f(\tilde{y}_m) > 1/(y_{\text{max}} - y_0) \) (the limiting case is the uniform distribution). A nonegalitarian policymaker \([h(y) = y]\), evaluating (2) at \( \tilde{y}_m \) finds, by substitution, \( V'(\tilde{y}_m) < 0 \), so the income-maximizing standard, \( \tilde{y}^+ \), is less than \( \tilde{y}_m \). This holds a fortiori for egalitarian policymakers, \( \tilde{y}^* < \tilde{y}^+ < \tilde{y}_m \). Therefore, the median voter prefers a standard which is inefficiently high, as measured by any disinterested social welfare function, including that which might represent the median voter’s own philosophical principles.\(^{11}\) It is not clear which is worse: an overly egalitarian school system, bureaucratically insulated from the voters, which sets standards too low; or an empowered electorate which sets standards too high.

10. The classical condition of single-peakedness (Duncan Black, 1958) and its generalizations (Amartya K. Sen, 1966) do not hold here, since each voter is indifferent among standards his child will not meet, \( \tilde{y} > \tilde{y}_i \). The median-voter rule, therefore, does not hold in its strongest form, where \( \tilde{y}_m \) is strictly preferred by a majority to all alternatives, but it does hold in a weaker form (Michael Dummett and Robin Farquharson, 1961), where \( \tilde{y}_m \) is weakly preferred. Moreover, \( \tilde{y}_m \) is the unique standard to satisfy this condition. It can be shown that this will be a voting equilibrium if indifferent voters support either the existing standard or the lower of any two proposed standards (the one closer to their children’s upper limit). If indifferent voters abstain, then there is no voting equilibrium.

11. Some might interpret such election results as a victory for elitists, with convex \( h \) functions, and a defeat for egalitarians, with concave ones, but the results say nothing about the social welfare function preferred by the median voter. Parents of hard-working students will vote against egalitarian-sounding candidates and parents of other students will vote for such candidates, but all parents are simply voting their own self-interest regardless of whether it helps or hurts the interests of the other voters.
This result, for distributions which are symmetric unimodal, or nearly so, is consistent with the polling data. However, if the distribution is sufficiently skewed, tailing off to the right, the standard set by policymakers who are not overly egalitarian can be stricter than the median voter would like.

IV. Decentralization Lowers Standards and Welfare Among Identical Districts

It is often argued that lower standards in the United States, compared to our international competitors, are in part attributable to our decentralized educational system. There is a fairly straightforward free-rider problem here. Higher standards and productivity for a single district will not be fully appropriated by its graduates in higher wages, since most graduates seek employment elsewhere, and the district’s reputation will not completely follow them. Thus, decentralization reduces the district’s marginal benefit of a higher standard. Much of the benefit of higher productivity spills over to the graduates of other districts, who are pooled with them.

In addition, and perhaps less well understood, decentralization raises the marginal cost of a higher standard, since more students will choose not to meet it, lacking a full payoff. For both reasons, lower benefit and higher cost, a decentralized system results in lower standards and lower social welfare.

Suppose all school districts are alike.12 The wage earned by a district’s graduates is no longer simply \( w = \tilde{y} \), but is now a weighted average of the standard \( \tilde{y} \) and the average standard in other districts, \( \tilde{y} \):

\[
    w = (1-\theta)\tilde{y} + \theta \tilde{y}
\]

The pooling parameter \( \theta \) reflects graduate emigration and imperfect reputation effects of the district’s diploma (or possibly employer fear of EEOC action for distinguishing among diplomas).13 In the limiting case of no pooling (\( \theta = 0 \)), decentralized standards are the same as under centralization, since each district replicates the whole system, without spillovers.

Formally, each district’s distribution function is \( F(\hat{y}; \tilde{y}, \theta) \), representing those students for whom \( U'(L_0, y_0) \geq U'[\ell'(\hat{y}) \pm \theta(1-\theta)\tilde{y}] \). At symmetric configurations, where \( \hat{y} = \tilde{y} \), we have \( w = \tilde{y} \), and \( F(\hat{y}; \tilde{y}, \theta) = F(\hat{y}; \theta = 0) \), independent of \( \theta \). That is, the family of curves in \( \theta \), plotting \( F(\hat{y}; \tilde{y}, \theta) \) against \( \hat{y} \), all cross at \( \hat{y} = \tilde{y} \).

Consider two such curves, \( F'(\hat{y}; \tilde{y}, \theta_1) \) and \( F'(\hat{y}; \tilde{y}, \theta_2) \), where \( \theta_1 < \theta_2 \). For \( \hat{y} < \tilde{y} \), \( (1-\theta_1)\tilde{y} + \theta_1 \tilde{y} < (1-\theta_2)\tilde{y} + \theta_2 \tilde{y} \), so there are more students for whom \( U'(L_0, y_0) \geq U'[\ell'(\hat{y}) \pm \theta(1-\theta)\tilde{y}] \) than for whom \( U'(L_0, y_0) \geq U'[\ell'(\hat{y}) \pm (1-\theta_2)\tilde{y} + \theta_2 \tilde{y}] \). The converse holds for \( \hat{y} > \tilde{y} \). That is, for \( \theta_1 < \theta_2 \), \( F'(\hat{y}; \tilde{y}, \theta_1) \geq F'(\hat{y}; \tilde{y}, \theta_2) \) as \( \hat{y} \approx \tilde{y} \) [i.e. \( F'(\hat{y}; \tilde{y}, \theta_1) \) cuts \( F'(\hat{y}; \tilde{y}, \theta_2) \) from above at \( \hat{y} = \tilde{y} \)]. Thus, \( F'(\hat{y}; \tilde{y}; \theta_1) \leq F'(\hat{y}; \tilde{y}; \theta_2) \), and \( dF'(\hat{y}; \tilde{y}; \theta) / d\theta \geq 0 \), typically with strict inequality. This shows that with pooling (\( \theta > 0 \)) decentralization leads a rise in the standard to deter more students than under centralization, as stated above.

Each district chooses \( \hat{y} \) to maximize its objective function,

\[
    V(\hat{y}; \tilde{y}, \theta) = \left[ 1 - F(\hat{y}; \tilde{y}, \theta) \right] h \left[ (1-\theta)\tilde{y} + \theta \tilde{y} \right] + F(\hat{y}; \tilde{y}, \theta) h(y_0)
\]

there is among the non-college-bound, particularly if employers do not distinguish among diplomas. On the other hand, if employers do not distinguish among diplomas, then the model gives unrealistically high values of \( \theta \) for typical rates of graduate mobility. Formally, it can be shown that

\[
    \theta = 1 - a^2 - (a^2 - (n-1)/n)
\]

where \( a \) is the proportion of graduates remaining in each of the \( n \) districts. For example, Burton A. Weisbrod’s (1964) case study of Clayton, Missouri, finds educational capital exported and imported at a rate of 90 percent, which would imply \( \theta \) of about 0.99 for large \( n \). However, most of the migration is to nearby districts (in Weisbrod’s study, over half is intracounty), so it is reasonable to assume that employers have some information about various diplomas and place some weight on that, lowering \( \theta \).
for given \( \bar{y} \). The solution to the model is found by evaluating the district's first-order condition at symmetric equilibrium, \( y = \hat{y}^* \):

\[
V'(\hat{y}^*; \bar{y} = \hat{y}^*, \theta) = (1 - \theta)[1 - F(\hat{y}^*; \theta = 0)]h'(\hat{y}^*) - F'(\hat{y}^*; \bar{y} = \hat{y}^*, \theta) \times [h(\hat{y}^*) - h(y_0)] = 0.
\]

In examining comparative statics, the second-order condition and a stability argument can be used to show that \( \text{sgn} \frac{d\hat{y}^*}{d\theta} = \text{sgn} \frac{dV'}{d\theta} \). This is negative, so with pooling (\( \theta > 0 \)) decentralization lowers standards below the centralized level (equivalent to \( \theta = 0 \)). The two reasons given above for this result are identifiable in (11). The first term is reduced by a positive \( \theta \), reflecting the district's attenuated marginal benefit of a higher standard (some of the benefits go to graduates of other districts, by raising their \( \bar{y} \)'s). The increased marginal cost of a higher standard is given in the second term, since a positive \( \theta \) raises \( F'(\hat{y}^*; \bar{y} = \hat{y}^*, \theta) \), as more students decide that the incomplete payoff is not worth the effort.

This result may be helpful in understanding why standards are lower in the United States than abroad.\(^{14}\) It also illustrates the benefits (though not the costs) of the proposed program of national standards and testing. All districts (assumed to be identical) gain unambiguously, as centralization raises \( \hat{y} \) and \( \bar{y} \) pari passu, increasing the value of the common objective function, \( V \). For the remainder of this paper, I return to the basic model of a centralized policymaker (or, equivalently, \( \theta = 0 \)).

V. Comparative Statics in the Basic Model

A. Shifts in Student Preference for Leisure

Consider an increase in student preference for leisure.\(^{15}\) In terms of Figure 2, let the indifference curve through \( (L_0, y_0) \) pivot up for some or all students. Obviously, this would reduce the number of students meeting any given standard. But does this lead the policymaker to acquiesce in a lower standard? Perhaps surprisingly, the answer is "Not necessarily."

The reason for the ambiguity can be ascertained by examining the first-order condition (2) in the basic model. An increased preference for leisure reduces the benefit of raising the standard, the first term, since the number of graduates at the old optimum, \( [1 - F(\hat{y}^*)] \), declines. This inclines policymakers to acquiesce in a lower standard, as most observers would expect.

However, the cost of raising the standard, the second term, may rise or fall, depending on whether the increased preference for leisure brings more students or fewer students onto the margin (i.e., whether \( f(\hat{y}^*) \) rises or falls). This depends on whether the number of students who were previously on the margin, and now drop below it, are sufficiently replenished by those who were above the margin and now drop down to it.

The case of a uniform leftward shift of the distribution (and density) function is shown in Figure 3. The distribution function is parameterized by \( \lambda \) in an additive fashion, \( F(\bar{y}_{\lambda} + \lambda) \). A marginal rise in \( \lambda \) reduces the maximum standard that any student will meet and reduces the number of graduates at the old optimum, as just discussed. The effect on the number of marginal students is simply \( f'(\hat{y}^*) \), the slope of the density function at the old optimum. If the optimal

\(^{14}\) Interestingly, in a median-voter model, it can be shown that decentralization has no effect on the equilibrium standard, provided \( \theta < 1 \).

\(^{15}\) "Leisure" here refers to nonstudy activity, including part-time employment. Also, shifts in preferences need not originate with the student. For example, changes in family structure might reduce the student's nonpecuniary cost of leisure if no one is making him or her do the homework.
standard lies on a rising portion of the density function, then a shift to leisure brings more students onto the margin, and the optimal policy is to acquiesce in a lower standard. This will hold for the benchmark case of symmetric, unimodal density functions, since, as has been seen, \( \tilde{y}^* < \tilde{y}_m \), so \( f'(\tilde{y}^*) > 0 \). If the distribution is asymmetric, however, this result need not hold.

Formally, differentiating the first-order condition (2) gives

\[
\frac{d\tilde{y}^*}{d\lambda} = \frac{f'(1-F) + f^2}{f'(1-F) + 2f^2 - (1-F)f\tilde{h}''/\tilde{h}'}
\]

where the denominator must be positive in order to satisfy the second-order condition. Consider an asymmetric example where \( F \) can be described over the relevant range by the class of exponential distributions, with \( f' < 0 \) throughout: \( F(\tilde{y}_i + \lambda) = 1 - \exp[-g(\tilde{y}_i + \lambda)] \), \( g' > 0 \), \( g(y_0) = 0 \). Evaluating (12), then

\[
(12') \quad \text{sgn} \frac{d\tilde{y}^*}{d\lambda} = -\text{sgn} g''(\tilde{y}^* + \lambda).
\]

For the simple exponential function, with linear \( g \), \( \tilde{y}^* \) is invariant with respect to \( \lambda \). However, for \( g'' < 0 \), \( f' \) is more negative, and the optimal response is to raise standards, not to acquiesce in the lower standard sought by the marginal students. Here,

the policymaker finds there are enough fewer students on the margin to write them off and to raise the standard for those who will meet it.

B. Neutral Shifts in the Educational Production Function

Now consider a deterioration of nonstudent inputs to the educational production function, such as teacher or school quality, or home educational inputs. This is a shift down of \( y'(L_i) \) in Figure 2. As with a shift in student preferences toward leisure, this reduces the number of students meeting any given standard, while increasing or decreasing the number of marginal students. However, there is an additional consideration here, the possible effect on zero-effort productivity. This makes a deterioration of the production function more likely to lead to lower standards than a comparable shift in student preferences.

Specifically, consider a “neutral,” or parallel shift down, of \( y'(L_i) \), coupled with quasi-linear preferences (vertically parallel). A downward shift of \( \lambda \) units reduces the maximum standard any student will meet by the same amount, so the distribution function \( F(\tilde{y}_i) \) shifts left as in Figure 3. However, the shift is not truncated: \( y_0 \) also falls by \( \lambda \). Differentiating the first-order condition (2) now yields

\[
(13) \quad \frac{d\tilde{y}^*}{d\lambda} = -\frac{f'(1-F)+f^2[1+h'(y_0)/h'(\tilde{y}^*)]}{f'(1-F)+2f^2-(1-F)f\tilde{h}''/\tilde{h}'+h'(\tilde{y}^*)/h'(\tilde{y}^*)}.
\]

By comparison with (12), one immediately sees that the drop in \( y_0 \) reduces the optimal standard compared to the truncated case of shifting preferences. Specifically, (12) implies \( \frac{d\tilde{y}^*}{d\lambda} > -1 \) for any objective function: the standard drops, but not by enough to keep the graduation rate from falling. By contrast, for the neutral production-function shift, (13) shows that in the GDP-maximizing case (\( h' = 1 \)), \( \frac{d\tilde{y}^*}{d\lambda} = -1 \): the
standard drops by the full amount of the shift, and the optimal graduation rate is unchanged. For some moderately egalitarian examples, the optimal standard falls by more than the shift, \( d\hat{y}^*/d\lambda < -1 \). Here, egalitarian concern with the worsened condition of nongraduates leads policymakers actually to raise the graduation rate, by greatly reducing standards.

To summarize, a neutral downward shift of the production function is much like a shift in student preferences toward leisure, but it also raises the social cost of nongraduation, for any given standard. This leads policymakers to choose a lower standard than otherwise. I could not prove that the standard necessarily declines, but I could find no counterexample, either. Egalitarian policymakers may even choose to lower standards so much that the graduation rate rises.

C. Neutral Shifts in the Demand for Skills

The function \( y'(L_i) \) is not simply an educational production function, but also reflects the market demand for skills. Properly speaking, it is a composite function \( y'(L_i) = y(s'(L_i)) \), where \( s'(L_i) \) is the educational production function, generating skills from student input, and \( y(s) \) is the market demand function for skills. Neutral technical progress shifts up the function \( y'(L_i) \) and thereby shifts up the function \( y'(L_i) \) in Figure 2. In the case of quasi-linear preferences, one can just reverse the analysis of the previous section, though now one must distinguish between changes in the market and changes in skills.

For example, in the GDP-maximizing case, the wage of graduates rises by the full amount of the upward shift, but this represents only the increased demand for skills, with no change in the standard of skills required for graduation. By contrast, for those egalitarian cases mentioned above in which the graduate wage changes by more than the shift of the production function, one would observe a rise in skill standards: here, the egalitarian concern for nongraduates is attenuated by the rise in nongraduate wages.

As a final and possibly relevant example, consider a simultaneous shift up of the demand for skills and an equivalent shift down of the educational production function (due to diminished nonstudent inputs). Obviously, each student is just willing to meet the same standard of productivity as before (regardless of preferences), so the distribution function \( F(\hat{y}_i) \) is unchanged; so is the optimal standard, in terms of the graduate wage, \( \hat{y}^* \). However, this represents the offsetting effects of increased demand for skills and lower skills required for graduation. That is, the skills standard declines (though it takes the same student effort to meet it, with diminished home and school inputs), the graduate wage is unchanged, and so is the graduation rate.

D. Biased Shift in the Demand for Skills

Now consider a biased shift in the demand for skills, against lower-skilled workers (e.g., due to biased technical progress or factor-price equalization). This shifts down the lower end of the \( y'(L_i) \) function, in the vicinity of \( y_0 \). To some, this adverse effect on nongraduates makes it all the more important to reduce the nongraduation rate, which means lower standards. However, there is a conflicting influence on the optimal standard, since the drop in demand for nongraduates gives students a greater incentive to meet the standard.

Formally, the adverse effect on nongraduates of a drop in \( y_0 \) is reflected in the second term of (2), raising the cost of high standards. On the other hand, Figure 2 shows that a drop in \( y'(L_i) \) around \( y_0 \) raises the standard any student is willing to meet, \( \hat{y}_i \), so \( F(\hat{y}_i) \) falls. In terms of equation (2), the benefit of a high standard (the first term) rises, since more students will meet it out of fear of the deteriorated alternative.

Which factors lead one effect or the other to prevail? One factor is the degree of egalitarianism. More egalitarian policymakers will obviously give greater weight to the adverse effects on nongraduates, the first effect, and would therefore be more inclined to reduce standards in response to a drop in \( y_0 \).
Another factor is student homogeneity, which tends to reduce the number of non-graduates to worry about, and thus favors higher standards. To take an extreme case, if students are perfectly homogeneous, the standard will simply be set at everyone’s identical $\bar{y}$. A drop in $y_0$ raises this standard unambiguously. Obviously, if the population is more heterogeneous, there will be some nongraduates, so policymakers will be less inclined to raise standards.

E. Shifts in Student Heterogeneity

As I have shown, the degree of student heterogeneity can affect the sign of policymaker response to exogenous shifts. But what about the effect of heterogeneity itself? Consider the case of a symmetric unimodal distribution, where heterogeneity (in either student preferences or educational production functions) increases the spread around an unchanging mode. Beginning with zero heterogeneity, then obviously the optimal standard is the mode, where all the students are concentrated. It immediately follows that the introduction of heterogeneity reduces the optimal standard, since it was shown above that for nondegenerate symmetric unimodal distributions, the optimal standard is below the mode. This holds for any degree of egalitarianism of the policymaker.

However, as heterogeneity increases further, it may lead standards to rise again, depending on the policymakers’ degree of egalitarianism. Consider an example, where $\bar{y}$ follows a symmetric beta distribution on the interval $(1,2)$. The optimal standard $\hat{y}^*$ is shown in Figure 4 for two constant-elasticity-of-substitution (CES) social welfare functions: GDP-maximization ($\sigma \to \infty$) and a rather egalitarian one ($\sigma = 0.1$). As heterogeneity is introduced, the optimal standard falls, for any social welfare function, as stated above. However, for low degrees of egalitarianism, the optimal standard rises again, as heterogeneity increases further.

Curiously, this result does not reflect the nonegalitarian’s greater weight attached to graduates, per se. For both policymakers, the number of graduates at the optimal standard $\hat{y}^*$ declines with increased heterogeneity (the lower tail $F(\hat{y}^*)$ grows in size), reducing the benefit of high standards for both of them [the first term in (2)]. The greater weight attached to graduate fortunes by the nonegalitarian strengthens this reason to lower standards, instead of explaining the decision to raise standards.

What distinguishes the two policymakers is that they are responding to different changes in the number of marginal students, $f(\hat{y}^*)$. An increase in heterogeneity reduces $f$ in the region just below the mode, but it raises $f$ farther down in the tail. As shown in Section I, the less egalitarian standard will be higher, so it will be closer to the mode. That means the number of marginal students will be more likely to drop with heterogeneity in the less egalitarian case, leading to a drop in the cost of

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16This distribution can be shown to characterize a student population which shares the same Cobb-Douglas production function, $y(L) = y_0 + (L_0 - L)^{0.5}$, where $y_0, L_0 = 1$, and the same family of utility functions $U(L, w) = [(1 - \gamma)L^{-1} + \gamma w^{-1}]^{-1}$ (i.e., CES with elasticity of substitution equal to 0.5). Student preferences vary by weights attached to leisure and future income, with $\gamma$ following the symmetric beta distribution on the interval $(0,1)$. 
high standards [the second term in (2)]. Again, the fact that the nonegalitarian attaches less weight to the losses of these marginal students is not the immediate driving factor. Rather, the nonegalitarian raises the standard because he or she is operating in a region where the number of marginal students to be concerned about drops.

VI. Perfect Information versus Pass/Fail Standards

This model of educational standards rests on the observation that employers receive only coarse information regarding high-school graduates. Bishop (1988, 1989, 1990) argues this is the underlying explanation of low achievement, with its attendant cost in productivity growth. He advocates a host of measures to improve information flows, ranging from statewide testing to distributing transcripts enclosed in plastic, in order to raise the average level of achievement. This can also advance egalitarian goals, by giving students less extreme alternatives to dropping out, as John D. Owen (1994) points out. By contrast, much of the thrust of recent affirmative action has focused on distinguishing “qualified” from “nonqualified” job candidates (a binary credential), rather than choosing the “best qualified” candidate (a continuous measure).

Perfect information is the world of Figure 1, where every student chooses his or her own leisure-wage configuration. Educational standards, in the sense I have modeled them, have no meaning or relevance here: policymakers have no standard-setting function. Would perfect information raise GDP or social welfare over that engineered by standard-setters? The answers depend on the degrees of student dispersion, policymaker egalitarianism, and pooling under decentralization. Perfect information raises effort and achievement for students in both tails of the distribution but reduces them for those in the middle. Those at the bottom, who choose zero effort, would typically be willing to exert some effort, just not as much as the standard. Those at the top, the highly motivated students who find no payoff in exceeding the standard, would also choose higher effort with perfect information. However, between the tails are the marginal and near-marginal graduates, who reluctantly meet the standard, and who would choose a lower effort level with perfect information. Therein lies the ambiguity.

In the limiting case of zero student dispersion, no students are in the tails, and perfect information reduces achievement (and social welfare). Any policymaker who placed no value on student leisure would choose \( \tilde{y} \), in Figure 2, pushing each student to his or her limit. Perfect information leads all students to choose less effort, at the tangency depicted in Figure 1. This result is independent of the degrees of egalitarianism and pooling under decentralization.

Once some dispersion is introduced, the policymakers’ standard depends on the degree of egalitarianism. Rawlsian policymakers choose very low standards, so perfect information would definitely raise GDP once there is any dispersion at all. Income-maximizing policymakers, however, might still be generating higher GDP than would perfect information, provided the degree of dispersion is small. At higher degrees of dispersion, perfect information would raise GDP above that engineered by any standard-setter, including the income-maximizer.

The effects of perfect information are more salutary in a decentralized system, since the standards are even lower than the policymakers would like. With decentralization, it takes less dispersion and less egalitarianism for perfect information to raise GDP.

Typically, perfect information affects the value of the policymakers’ objective function, \( V \), in the same direction as GDP. If so, the incentives for policymakers to facilitate or suppress perfect information are compatible with the goal of raising GDP, even if

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17The patterns and possibilities discussed here are illustrated by simulation results in my working paper (Costrell, 1992).
they are rather egalitarian. In some cases, however, egalitarian policymakers have an incentive to suppress perfect information even when it would raise GDP, and in other cases, egalitarian policymakers would facilitate perfect information, even though it reduces GDP.

In sum, the case for perfect information would appear to be strong, if not airtight: for most plausible degrees of heterogeneity, egalitarianism, and pooling under decentralization, perfect information not only raises GDP, but also social welfare.

VII. Conclusions

The basic model of a centralized standard-setter can be interpreted normatively, as a model of the optimal standard, provided the social welfare function is properly chosen. In this model, if strong student preferences for leisure or low nonstudent inputs lead to low standards (which need not be the case), then social welfare is reduced, not improved; by simply raising standards. Policies should be directed instead at the exogenous shifts (e.g., policies to facilitate family cohesion).

Decentralized standards, however, will be suboptimal under pooling with identical districts, in which case there are benefits in moving toward a national standard at a higher level. Standards will also be too low if the standard-setters are more egalitarian than the “true” social welfare function. The strategies for raising standards here depend on why standard-setters are so egalitarian. If, for example, excess egalitarianism derives from the prevailing culture at many of the nation’s education schools, then alternative certification might help.

I have also shown that standards can be raised by measures to improve accountability to voters. In the case of symmetric unimodal distributions, the median voter will want to go too far, beyond the optimal standard. However, even so, median-voter rule can be an improvement if standard-setters are excessively egalitarian or decentralized.

Finally, even if standards are chosen optimally, it may be better yet to improve information flows such that standards become less important. With high student dispersion, this will typically improve social welfare.

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