Accounting for the rise in unfunded public pension liabilities: faulty counterfactuals and the allure of simple gain/loss summations*

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Abstract

This paper provides a methodological critique of an influential method for assessing the impact on the Unfunded Accrued Liabilities (UAL) of the gap between assumed and actual investment returns over extended periods, and offers a sound replacement. The method in question simply sums over time the components of the annual actuarial gain/loss report. This implicitly assumes that in the counterfactual exercise, the interest on the additional UAL is covered dollar-for-dollar by amortization. But under actual funding formulas amortization usually varies less than interest. This means there are large intertemporal interactions between the gap in investment returns and subsequent shortfalls between contributions and interest. Using the actual funding formula in the counterfactual can lead to much higher estimates of the UAL impact of the gap in investment returns because it does not assume away these interactions. This method can more accurately inform policy-makers, regarding the importance of cutting the assumed rate of return.

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1 Introduction and summary

What has been the impact on public pension funds' Unfunded Accrued Liabilities (UAL) of the gap between assumed and actual investment returns, over extended periods? How has this impact compared with that of other factors that raise the UAL? An influential[1] approach to this question (Munnell *et al.*, 2015) takes the components of the actuarial gain/loss statement from a plan's annual valuation reports, and simply

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[1] For example, the State of Connecticut recently commissioned the Center for Retirement Research at Boston College to apply this method to the state's public pension history, and the Governor drew on the findings to form his recommendations (Aubry and Munnell, 2015; Malloy, 2015, slide 22).

adds them up over time. In this paper, I critique the methodological basis for this procedure and propose a sound replacement. I find that the method in question significantly low-balls the impact of the gap in investment returns in many cases. The policy implication is to under-emphasize the need to cut the assumed return on investment.

The procedure's problem arises in the last step. Even if the *annual* gain/loss components of the rise in UAL are accurate, adding them up over a period of years to arrive at a cumulative breakdown of the *multi-year* rise in UAL is problematic. The reason is that this procedure, which I shall call the simple summation (SS) method, does not realistically model the subsequent amortization payments triggered by any rise in UAL. Instead of using the actual amortization policy, the method implicitly assumes amortization rises to cover the interest on the UAL.

For example, if the UAL rises by $100, due to the gap in investment returns, the SS method assumes the following year's amortization payments will rise by the interest on that $100, e.g., $8.00 under a typical assumed return of 8%. However, under the actual amortization formula commonly used ('level percent of payroll'), those payments will often rise by a lesser amount. With parameters of 4% payroll growth and 30-year amortization, the payment would be $5.90, or $2.10 short of covering the interest. Consequently, the initial $100 impact on the UAL will rise over a multi-year period with accrued unpaid interest. By analogy, the financial impact of a $100 credit card debt will grow over time if the user's policy is to not pay the full interest. In the public pension context, this will be the case when the amortization schedule is back-loaded, under policy choices that combine long amortization periods, high assumed payroll growth (which tilts the schedule up), and high assumed investment return (since that is the interest rate that accrues on the UAL).

By contrast, the SS method holds the initial $100 impact constant by assuming payments rise $8.00 to cover the additional interest, instead of following the actual amortization formula. The SS method attributes the subsequent rise in UAL from accrued unpaid interest ($2.10 in the first year) to contribution shortfalls instead of the gap in investment returns that triggered it. As a result, the SS method will often underestimate the multi-year impact of the gap in returns (or anything else that raises the UAL). As I will show, with the example of the Connecticut State Teachers' Retirement System, this underestimate can be substantial.

It is important to get the impact of the gap between assumed and actual returns right, not only for academic reasons, but also for practical policy purposes. In the aftermath of the 2007–2009 market crash, many funds reduced their assumed returns, but not by much (0.27%, on average, Biggs, 2015). Several years later, the cuts have in many cases proven insufficient to prevent a further rise in UALs. Thus, pension policy-makers – and the general public – need to understand the full impact of high assumed returns to better motivate rate-cutting policy going forward. By underestimating the past impact, the SS method can lead policy-makers to place less emphasis on reducing the assumed return than is warranted. At the same time, the important intertemporal interaction between the amortization policy and the UAL-drivers reinforces the need to keep the amortization period short and the assumed payroll growth low.

The underlying problem with the SS method is that it has emerged without careful attention to the counterfactual scenarios it purports to address, such as, 'What if investment returns had met assumptions, while other UAL-drivers remained unchanged?'[2] Under the SS method, the key 'other' UAL-driver is the contribution shortfall between interest on the UAL and amortization.[3] Thus, in the counterfactual for the gap in investment returns the contribution shortfall is held fixed (at its actual value). This is where the implicit assumption regarding amortization creeps in. If the contribution shortfall is held fixed, then amortization varies dollar-for-dollar with the interest on the counterfactual UAL. To restate the example given above more precisely, if the UAL had been $100 *lower* in this counterfactual (as assumed returns were met), the subsequent amortization would have been reduced by the full $8.00 interest on that $100, thereby attenuating the subsequent counterfactual drop in UAL. Thus, the difference between the actual UAL and the counterfactual UAL at the end of the period – the multi-year impact of the gap in investment returns – is underestimated. This is the crux of the problem.

The SS method also yields untenable results on other UAL-drivers. Most strikingly, under this method the cumulative UAL impact of pension obligation bonds (POBs) is no different from the initial impact of receiving the proceeds. If a state issues POBs to 'pay down,' say, $2 billion in a fund's UAL (as did Connecticut for its teachers' fund in 2008) the plan's annual gain/loss statement will record a $2 billion reduction in the UAL that year due to the POB. The problem is that in the SS multi-year analysis, the cumulative impact remains at $2 billion every year thereafter. There is no impact attributed to the POB of the return on the proceeds invested in the fund (let alone the outstanding POB debt incurred by the state). Indeed, even if the invested proceeds quickly lose substantial value – as happened in Connecticut – this will have no effect on the cumulative impact under the SS method. This is clearly misleading.

The structure of the paper is as follows. First, I review the analytics behind the SS method. Next, I dissect the key issue of how the method's counterfactual model of amortization diverges from actual amortization formulas, and draw its implications. I then illustrate my proposed method of using actual amortization formulas. To complement our understanding of the impact of UAL-drivers under the different amortization assumptions, I show that the difference is mirrored in the impact on cumulative contributions. The *total* impact – paid and not-yet-paid (amortization and UAL, respectively) – is essentially invariant. Finally, I show how the SS method provides an unsatisfactory treatment of POBs, due to its implicit amortization assumption, and that a correct treatment can qualitatively reverse the measured impact of the POBs on the UAL, as in the Connecticut case.

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[2] In assessing the impact of the gap between assumed and actual returns, this counterfactual treats actual returns as variable and assumed returns as fixed. As discussed further in the conclusion, the policy-relevant question would be the reverse. But it is the question asked by the SS method, so this paper examines whether it gives the right answer.

[3] Munnell *et al.* (2015) rightly emphasize that this is the UAL-relevant contribution shortfall, rather than the commonly cited shortfall between actual contributions and the ARC, as the ARC may not cover interest.

It is important to note that under my method, the sum of the UAL impact of individual drivers will *not* sum to the actual rise in the UAL, unlike the SS method. That is because of the intertemporal interactions that are recognized in my method, but effectively assumed away under the SS method. The important interaction here is between the gap in investment returns and the *actual* amortization formula. If the gap in returns raises UAL, but amortization does not cover the ensuing additional interest, then the UAL rises further – a result attributable to the gap in returns. If we separately consider the impact of payments that fail to cover interest, the accrued unpaid interest would appear in both impacts. Hence, the non-additivity. The SS method is additive, but only because, in calculating the impact of the gap in returns, it replaces the *actual* amortization formula with an artificial formula that covers the ensuing additional interest. The result is more aesthetic, but less policy-relevant. If the user's interest is the impact of the gap in investment returns, we should use the actual amortization formula to answer that question. Additivity is also not necessary to compare policies. Even if we cannot parcel out the rise in UAL as X percent from the gap in investment returns and Y percent from contribution shortfalls, we can still compare each factor's impact. The key policy implication of my method is that it will often show a much larger impact of the gap in investment returns, relative to contribution shortfalls, than does the SS method.

Throughout the paper, I supplement the analytical results with illustrations adapted from the Connecticut State Teachers' Retirement System (CSTRS), FY00-FY14. The CSTRS case is of interest in its own right for several reasons. The system's pension funding difficulties rival those of such well-known cases as California, New Jersey, Illinois and Pennsylvania. Unlike other systems, it did not reduce its assumed rate of return from 8.5% over this period. Finally, it has made use of POBs to 'pay down' its UAL. Thus, it is an interesting case to illustrate the effect of using a more sound method of assessing the impact of the gap between assumed and actual investment returns, as well as the impact of POBs.

2 The sources of the rise in UAL: the SS method

The starting point is the decomposition of the annual rise in UAL presented in a plan's valuation report. The sources of the rise can be categorized into: (i) the gap between assumed and actual investment returns; (ii) contribution shortfalls, between interest on the UAL and amortization payments; and (iii) adverse liability developments (changes in and deviations from actuarial assumptions, as well as benefit changes).[4]

Let us consider this decomposition formally, with some slightly simplified math:[5]

$$\mathrm{UAL}_t \equiv L_t - A_t \tag{1}$$

$$L_t = (L_t - L^e_{t|t-1}) + L^e_{t|t-1} = (L_t - L^e_{t|t-1}) + (1 + r^*)L_{t-1} + \mathrm{NC}_t - B_t \tag{2}$$

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[4] In a later section, I will add POBs to the analysis.
[5] I assume cash flows and annual liability accruals are made at the end of the year, rather than the usual actuarial assumption of mid-year, thus excluding the associated half-year interest on each.

$$A_t = (1 + r_t)A_{t-1} + \text{AMT}_t + \text{NC}_t - B_t, \tag{3}$$

where A_t = assets[6]; L_t = accrued liabilities; $L_{t|t-1}^e$ = expected accrued liabilities in period t, as of $t-1$, under actuarial assumptions; NC_t = normal cost; B_t = benefit payments; AMT_t = employer contributions in excess of NC, credited toward amortization; and r^*, r_t = assumed and actual return on investment.[7]

Substituting and simplifying, we have the standard actuarial gain/loss result, decomposing the change in the UAL from $t-1$ to t:

$$\Delta_{t-1,t}\text{UAL} = (r^* - r_t)A_{t-1} + (r^*\text{UAL}_{t-1} - \text{AMT}_t) + (L_t - L_{t|t-1}^e). \tag{4}$$

The first term on the right-hand side is the loss from the gap in investment returns; the second term is the loss from any failure of amortization to cover interest on the UAL; and the third term is the loss from liabilities exceeding expectations (for the reasons given above).

For the 1-year change in UAL, this is an attractive formulation: it unambiguously allocates ΔUAL among the gap in investment returns, contribution shortfalls, and liability losses in a way that adds up to the actual change in the UAL. There are no interactions to disrupt additivity. As shown below, interactions will arise from the lagged values of UAL and A in the multi-year ΔUAL, but for the 1-year change, these are pre-determined. Thus, each term in (4) answers a well-defined counterfactual question about factors that are independent of one another: 'How would the UAL in period t (and its rise from period $t-1$) differ from the actual if, in period t, investment assumptions had been met (or amortization had covered interest, or liability assumptions had been met)?'

How, then, to allocate the rise in UAL over multiple periods, from period 0 to T? The approach put forth by Munnell *et al.* (2015) at the Center for Retirement Research at Boston College is to simply add up each component of (4), year-by-year, to allocate the total change in the UAL among its various sources:

$$\Delta_{0,T}\text{UAL} = \sum_{t=1}^{T}(r^* - r_t)A_{t-1} + \sum_{t=1}^{T}(r^*\text{UAL}_{t-1} - \text{AMT}_t) + \sum_{t=1}^{T}(L_t - L_{t|t-1}^e)$$
$$= \Delta_{0,T}\text{UAL}^{r:\Sigma} + \Delta_{0,T}\text{UAL}^{C:\Sigma} + \Delta_{0,T}\text{UAL}^{L:\Sigma}. \tag{5}$$

The first line is correct: the rise in UAL from period 0 to T equals the T-period sum of each term in (4). The issue is the interpretation, term by term, in the second line. I label each term as the contribution to the rise in UAL from the three UAL-drivers, as attributed by the SS method: $\Delta_{0,T}\text{UAL}^{r:\Sigma}$, etc., where Σ denotes the simple summation from the annual gain/loss statements. (I use C in the second term to denote contribution shortfalls, $C_t \equiv r^*\text{UAL}_{t-1} - \text{AMT}_t$.) This interpretation purports to answer the T-period version of the questions given above: 'How would the UAL in period T (and its rise from period 0) differ from the actual if, in periods $1,\ldots,T$, investment assumptions (for example) had been met and the series of other drivers held at

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[6] These are market asset values, as are the return on assets, r_t. The implications of using smoothed asset values, while tangential to the subject of this paper, are briefly discussed in a note below.

[7] We take the assumed return r^* as constant, which is accurate for CSTRS for the period examined.
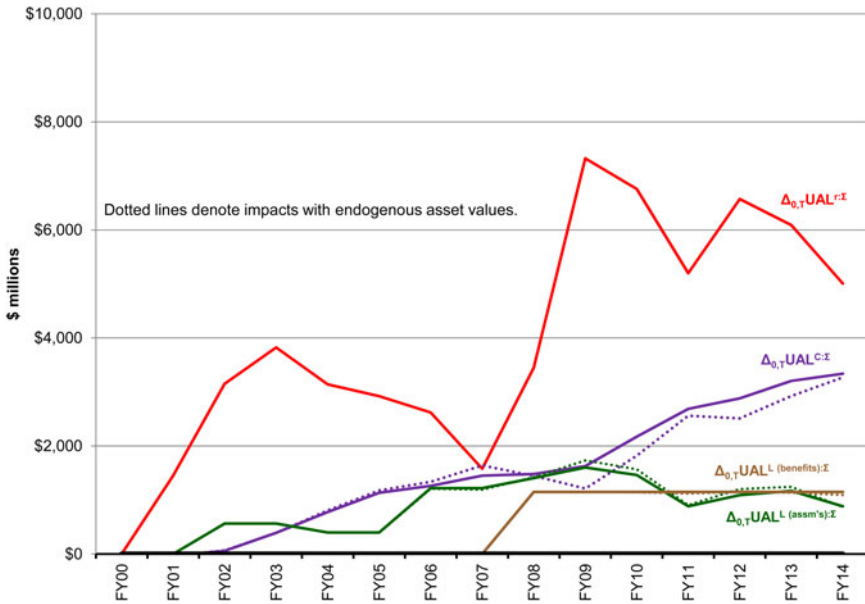
Figure 1. ΔUAL, Simple summation method: C_t exogenous. CSTRS, FY00–FY14 adapted from CSTRS: market asset values, without midyear return on cash flows and accruals.

their actual values?' Before exploring this question in depth, I illustrate the method's results with data from the CSTRS.

I use an adaptation of the CSTRS time series for FY00–FY14.[8] The main differences between this adaptation and the actual series are the use of market asset values rather than smoothed values and the exclusion here of the \$2 billion FY08 POB, to be considered separately below.[9] The net result of these adaptations is a rise in the UAL of \$10.4 billion from FY00 to FY14, rather close to the actual rise of \$10.6 billion (excluding the \$2 billion POB proceeds).

Figure 1 depicts the decomposition of the \$10.4 billion rise under the SS method. I find that this method attributes \$5.0 billion to the gap between the assumed investment return of 8.5% and actual investment returns (red line), $\Delta_{0,T}\text{UAL}^{r:\Sigma}$. Contribution shortfalls (purple line) account for another \$3.3 billion, $\Delta_{0,T}\text{UAL}^{C:\Sigma}$.[10] Thus, under this method, the impact of the gap in investment returns exceeds that of the contribution shortfalls by 50%.[11]

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[8] This study uses the series of biennial valuation reports from FY02–FY14.
[9] I also exclude the midyear return for cash flows and accruals from the growth in assets and liabilities, respectively – a relatively minor difference – to keep the math simple and focused on the issues at hand, and an FY08 actuarial adjustment for the interest on funds previously segregated for the COLA.
[10] The dotted lines will be discussed later.
[11] The allocation of the rise in UAL is sensitive to smoothing. In the CSTRS case, I find that smoothed asset values swing about 7 percentage points of the rise in UAL away from the impact of contribution shortfalls to the impact of the gap in investment returns. The reason is that over the period in question, smoothed asset values have, on average, exceeded market asset values for CSTRS by about 4%, and the smoothed UALs have, on average, been lower than the market UALs by about 7%. This means the interest on the UAL was also lower under smoothing, and, thus, the contribution shortfall from interest was

I have broken out liability losses into those due to actuarial assumptions and benefit enhancements, of which there was only one, in FY08.[12] In Fig. 1, note that the cumulative impact of the FY08 benefit hike, $\Delta_{0,T}\mathrm{UAL}^{L(\text{benefits}):\Sigma}$, is unchanged beyond FY08: the brown line is flat.[13] Indeed, under the SS method, the 1-year impact of *any* contributor to the rise in UAL, as specified in (4), is also the cumulative impact thereafter, in (5). Specifically, there is no further impact from the interest on the initial change in UAL. As explained below, that is because the method implicitly assumes amortization payments will be adjusted to cover the interest.

3 How does amortization behave in the implicit counterfactual?

Where does the implicit assumption about amortization come from? It is embedded in the counterfactual question underlying the SS method's decomposition in (5): 'How would the UAL in period T differ from the actual if investment assumptions (for example) had been met and *the series of other drivers held at their actual values*?' The counterfactual scenario has not been explicitly analyzed before,[14] but can be readily uncovered by comparing the actual $\Delta_{0,T}\mathrm{UAL}$ in (5) with the counterfactual $\Delta_{0,T}\mathrm{UAL}$ if investment assumptions had been met, $r_t = r^*$:

$$\Delta_{0,T}\mathrm{UAL} = \sum_{t=1}^{T} (r^* - r^*)A_{t-1} + \sum_{t=1}^{T} (r^*\mathrm{UAL}_{t-1} - \mathrm{AMT}_t)$$

$$+ \sum_{t=1}^{T} (L_t - L_{t|t-1}^e). \tag{5 CF^r}$$

Subtracting (5 CF^r) from (5), we get $\sum_{t=1}^{T} (r^* - r_t)A_{t-1}$, since the first term of (5 CF^r) is zero. This is $\Delta_{0,T}\mathrm{UAL}^{r:\Sigma}$, the SS method's attribution of the rise in UAL to the gap in investment returns, as given on the second line of (5). This is the procedure underlying the method's decomposition.

This procedure implicitly assumes the other two terms in (5) *and* (5 CF^r) *are equal*. Specifically, the series of contribution shortfalls and liability losses are held exogenous at their actual values while the gap in investment returns is assumed to vanish. This is certainly a defensible assumption for liability losses – there is no obvious link between them and the counterfactual investment returns.

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lower, too. Conversely, since smoothed asset values were higher, so was the assumed investment income, and, therefore, the gap between assumed and actual investment returns was greater. Although the asset-smoothing mechanism may well serve its designated purpose of smoothing out contributions, it can significantly distort our understanding of the rise in the UAL. Munnell *et al.*, use smoothed values, as embedded in the published gain/loss statements. In this paper, I use market values, both for the SS method and the alternative method proposed, to focus the comparison between the two on the fundamental difference, which is the treatment of amortization.

[12] The COLA, previously a fund-contingent benefit, was now guaranteed, adding \$1.15 billion to accrued liabilities. This effectively spent over half the \$2 billion proceeds of the POBs issued under the same legislative package.

[13] The same result can also be read off Table A5 of Aubry and Munnell (2015).

[14] The Munnell team's focus is on the categorization of terms in the gain/loss statement, and summing over time, rather than the underlying counterfactual (Munnell *et. al.* 2015, Table 1; Aubry and Munnell, 2015, Tables A3–A5).

The sticking point here is the implicit assumption that the counterfactual series of contribution shortfalls is the same as the actual, i.e., C_t is taken as exogenous under the SS method. Since the contribution shortfall is the gap between amortization payments and interest on the UAL, this means the counterfactual amortization must vary with the counterfactual interest to maintain C_t. That is, it is implicitly assumed that in the counterfactual with $r_t = r^*$,

$$\text{AMT}_t = r^*\text{UAL}_{t-1} - C_t, \tag{6}$$

where C_t is held constant at the actual level. Thus, in this counterfactual, where UAL_{t-1} is reduced from the actual, it is implicitly assumed that subsequent amortization would also be reduced, dollar-for-dollar with the interest on the UAL. (This assumption is also embedded in the counterfactual on liabilities.) The problem is this is *not* how amortization formulas work: amortization would commonly drop by *less* than the interest (although theoretically, it could drop by more, depending on the formula's parameters).

Specifically, the formula commonly employed is 'level percent' of payroll. With assumed payroll growth rate g, the basic form is:[15]

$$\text{AMT}_t = \text{UAL}_{t-1}(r^* - g)/\{1 - [(1+g)/(1+r^*)]^N\}, \tag{7}$$

where N is the remaining amortization period (closed or open). We can then write

$$\text{AMT}_t = \alpha_t(r^*\text{UAL}_{t-1}), \tag{8}$$

where $\alpha_t = [(r^* - g)/r^*]/\{1 - [(1+g)/(1+r^*)]^N\}$. The key points here, in comparison with (6), are: (i) amortization is proportional to the interest on the UAL, so there is no constant term as in (6) ($-C_t$); and (ii) the coefficient on the interest, α_t, can be less (or greater) than one, unlike (6). Indeed, if $C_t > 0$ (interest exceeds amortization, as is true on average, according to Munnell *et al.*, 2015), then α_t *must* be less than one. In other words, if there is a contribution shortfall under a proportional amortization formula, that directly implies amortization varies *less* than dollar-for-dollar with interest on the UAL, contrary to the implicit assumption of the SS method. This is the crux of the problem with the method – the main reason SS underestimates the impact of the gap in investment returns, as explained below.

The difference between (6) and (8) is illustrated in Fig. 2, which depicts counterfactual values of AMT_t on the vertical axis and counterfactual values of $r^*\text{UAL}_{t-1}$ on the horizontal. Equation (6) is parallel to the 45° line ($\text{AMT}_t = r^*\text{UAL}_{t-1}$), shifted down by a constant equal to the actual contribution shortfall, C_t. Equation (8) is a clockwise rotation of (6), through their common point, the actual value of ($r^*\text{UAL}_{t-1}$, AMT_t). This shows visually that if there is a contribution shortfall ($C_t > 0$) under a proportional amortization formula, the marginal response of amortization to a change in the interest on UAL (the slope of (8)) must be less than one (the slope of (6)), as assumed under the SS method.

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[15] As before, I abstract from actuarial conventions regarding mid-year cash flows. Another difference is that AMT is based here on UAL lagged 1 year, instead of 2 (or more), as occasioned by the timing of actuarial reports.
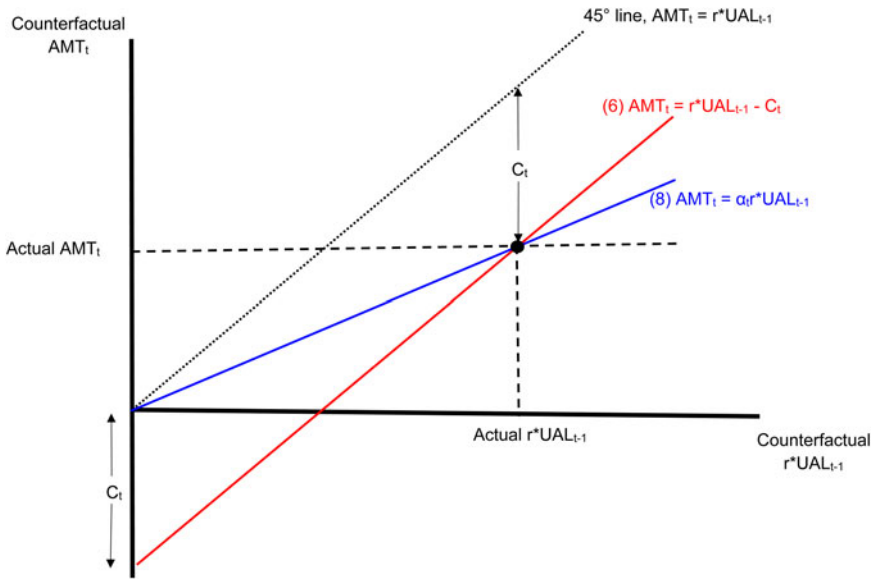
Figure 2. Counterfactual AMT_t as function of counterfactual r^*UAL_{t-1} under exogenous C_t, and exogenous α_t.

What policy variables, under control of the pension fund, generate this condition? That is, what combinations of r^*, g, and N result in $\alpha_t < 1$ in (8)? If the payment scheduled is back-loaded, payments may not cover interest. This will be true for combinations of long amortization periods, high assumed payroll growth (since high g lowers the initial payments)[16] and high assumed returns (since the interest is r^*). Formally, for any given value of N, we can derive a level curve for α_t in (r^*, g), above which $\alpha_t < 1$, as depicted in Fig. 3. For example, at the common values of $N = 30$ and $r^* = 0.075$, any $g > 0.013$ will generate contribution shortfalls. Calculating (8) for those plans in the BC Public Plans database that are coded as 'level percent' and which report g, I find almost all of them are characterized by $\alpha_t < 1$, with $r^* \geq 0.075$ and $g \geq 0.03$.[17] My proposed alternative to the SS method replaces the implicit assumption that C_t is exogenous (so $\alpha_t = 1$) with the assumption that α_t is exogenous at the actual observed value.

4 What are the implications of taking C_t as exogenous?

Before spelling out my alternative, I can sketch the implications of the SS method's treatment of C_t as exogenous. The main implication is that the SS method will underestimate the impact of the gap in investment returns if $\alpha_t < 1$, (and, conversely,

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[16] At $g = 0$ ('level dollar' amortization), $\alpha_t > 1$ for any values of r^* and N. But as $g \to r^*$, we can use L'Hôpital's Rule to show that $\alpha_t \to (1 + r^*)/r^*N$, so if $N > (1 + r^*)/r^*$, there will be a critical value of g, beyond which $\alpha_t < 1$.

[17] Of 160 plans in FY13 (the most current year with sufficient data on g), 103 are coded as 'level percent' (accounting for 63% of the total UAL) and of those, 36 plans report g (accounting for 59% of the UAL among 'level percent' plans). I calculate $\alpha_t < 1$ for 33 of those 36 plans, accounting for 95% of their UAL.
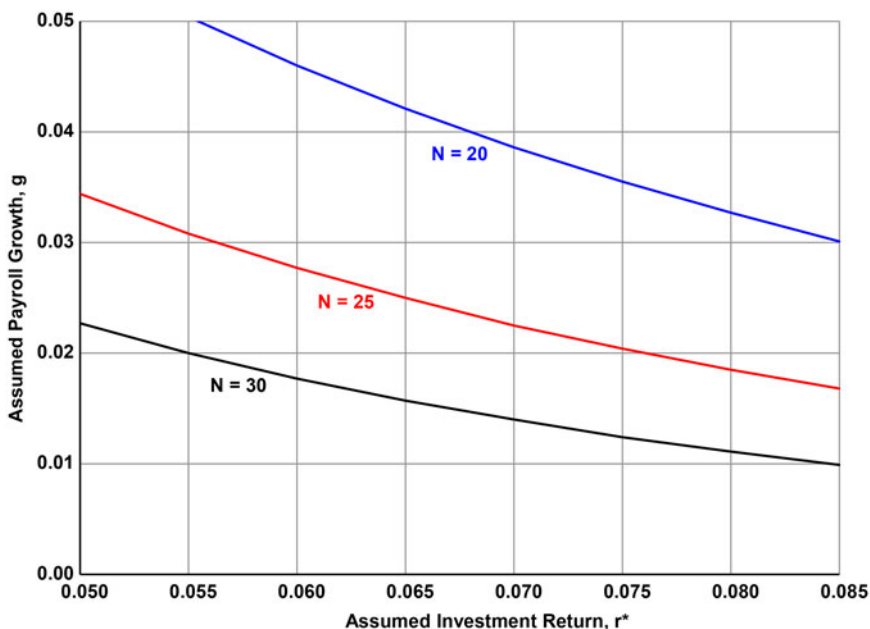
Figure 3. Level curves of g and r^* for $\alpha = 1$ (above the curves, $\alpha < 1$).

if $\alpha_t > 1$). The reason is that under the SS method, if the gap in returns generates an increment to the UAL and to the interest on the UAL the following year, amortization is implicitly assumed to cover the incremental interest, adding nothing to the contribution shortfall. But if the actual amortization process only partially covers additional interest ($\alpha_t < 1$), there will be an additional contribution shortfall, induced by the gap in investment returns.

This can be understood in terms of equation (5). Under the SS method, the gap in investment returns raises the UAL directly through the second term of (5), $\sum_{t=1}^{T} (r^* - r_t)A_{t-1}$, with no indirect impact through the third term, $\sum_{t=1}^{T} (r^*UAL_{t-1} - AMT_t)$, since it remains unchanged by implicit assumption that C_t is exogenous. But if amortization is actually governed by (8) with $\alpha_t < 1$, there will be an additional contribution shortfall induced by the gap in investment returns, reflected in the third term of (5). In short, there is a significant intertemporal interaction between the gap in returns and an amortization regime that generates contribution shortfalls.

The SS method's treatment of C_t as exogenous also explains the puzzling time pattern of other drivers' impact. Consider CSTRS' FY08 enhancement to the cost-of-living adjustments (COLA) that raised the UAL by \$1.15 billion. As Fig. 1 shows (brown line), under the SS method, the cumulative impact remains unchanged thereafter. There is no further measured impact from the accumulation of interest on the additional UAL because of the implicit assumption that additional amortization covers that interest. If, instead, the amortization regime does not fully cover additional interest ($\alpha_t < 1$), there would be additional contribution shortfalls, induced by the

benefit enhancement. The SS method assumes away the interaction between the benefit enhancement (or any other liability loss) and an amortization regime that underfunds the interest on the UAL.

5 Endogenizing asset values, for consistent counterfactuals

There is another problem with the SS method that needs to be fixed, before presenting the full alternative. That is the exogenous treatment of asset values in the counterfactuals for liability losses and contribution shortfalls. In constructing the counterfactuals underlying $\Delta_{0,T}\text{UAL}^{L:\Sigma}$ and $\Delta_{0,T}\text{UAL}^{C:\Sigma}$ in (5), the term $\sum_{t=1}^{T}(r^* - r_t)A_{t-1}$ is taken to be the same in the counterfactual as in the actual. That means the series of asset values A_t is taken as exogenous, at their actual values.[18] However, this is inconsistent for these counterfactuals, because if the liability losses (or contribution shortfalls) were absent, the UALs would be lower, and so would the amortization payments (under either (6) or (8)). Therefore, the asset series under these counterfactuals would differ from the actual asset series – A_t is endogenous.

The problem is solved by modeling the counterfactual values of A_t using (3) and embedding the counterfactual values of AMT_t. For the moment, I maintain the SS method's implicit assumption of exogenous contribution shortfalls, C_t, so counterfactual AMT_t follows regime (6). That is, internal consistency is satisfied by modeling the counterfactual values of A_t using (3) and (6), with the counterfactual series UAL_t that is implied by the exercise.[19]

How do the UAL-impacts under these consistent counterfactuals, endogenizing A_t, compare with the uncorrected SS method? The UAL-impacts of liability losses and contribution shortfalls now include the difference between $\sum_{t=1}^{T}(r^* - r_t)A_{t-1}$ calculated with actual and counterfactual asset values, instead of cancelling out. This is depicted in Fig. 1 for CSTRS as the difference between the solid and dotted curves. This is nearly imperceptible for the liability curves and is also very slight for the contribution shortfalls. Note also that endogenizing A_t has no effect on the impact of the gap in investment returns:[20] there is no dotted red line distinct from the solid line for $\Delta_{0,T}\text{UAL}^{r:\Sigma}$. For the other UAL drivers, I consider the dotted lines to be the correct impact of the UAL drivers, under the maintained assumption that C_t is exogenous.

Although the effect of endogenizing A_t is minor, it will be more salient when we consider POBs below. In any case, it is important to make the counterfactuals internally consistent, so in the remainder of this paper, I do so by treating A_t as endogenous.

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[18] The origin of this problem is that the setup of the SS method in (5) treats as exogenous the *dollar* value of the investment loss, $(r^* - r_t)A_{t-1}$, while the intent is surely to treat the gap in investment *returns*, $(r^*{-}r_t)$, as exogenous.

[19] I hold $(\text{NC} - \text{B})_t$ to their actual values for all of the counterfactuals. This would not be strictly valid for counterfactuals where benefits differ from the actual, but if benefit changes are minor – as in CSTRS over the period in question – this assumption should not be terribly problematic. Note also that I am assuming the difference $(\text{NC} - \text{B})_t$ is fixed at its actual value, a slightly weaker assumption than holding NC_t and B_t fixed separately.

[20] That impact, we saw above, is the difference between (5) and (5 CF^r). The values of A_{t-1} in (5 CF^r) should be endogenous, but since they are each multiplied by zero $(r^*{-}r^*)$, this makes no difference.
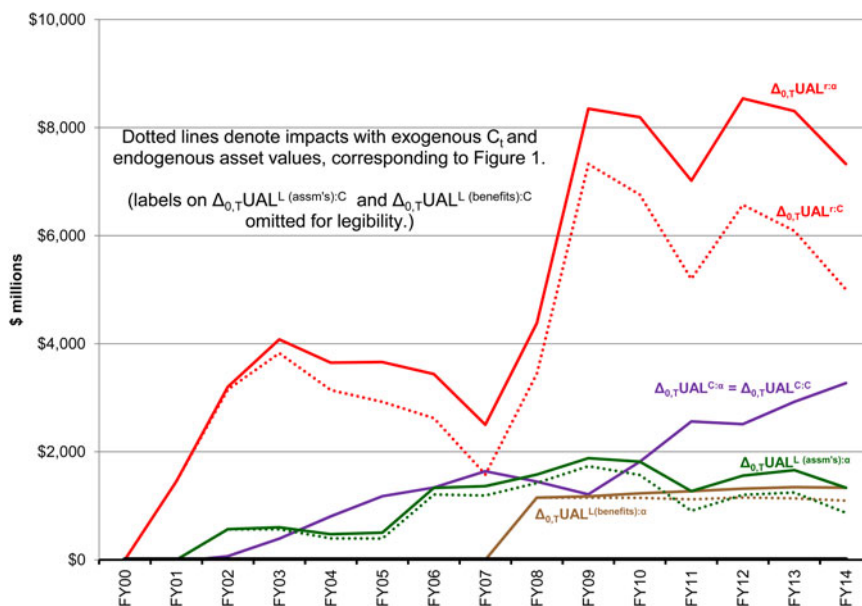
Figure 4. ΔUAL. α_t vs. C_t exogenous. CSTRS FY00–FY14. Adapted from CSTRS. Consistent counterfactuals.

We now return to the main problem with the SS method, the exogeneity of C_t and the treatment of amortization.

6 Exogenous amortization factor, α_t, vs. exogenous contribution shortfall, C_t

Instead of taking C_t as exogenous at its observed dollar levels (implicitly assuming $\alpha_t = 1$), I propose taking α_t as exogenous at its observed level (assuming proportional amortization), AMT_t/r^*UAL_{t-1}. Thus, I calculate the impact of the gap in investment returns by comparing the actual series UAL_t with the counterfactual UAL series generated by the system (1) – (3), (8), setting investment returns $r_t = r^*$, and holding the series α_t and L_t exogenous at their actual values. Figure 4, based on the data from CSTRS, compares the result with the impact calculated under exogenous C. The impact of the gap in investment returns (superscript notation $r{:}\alpha$, to denote exogenous α_t) is \$7.3 billion instead of \$5.0 billion under the SS method (notation $r{:}C$, to denote exogenous C_t).[21] This is a substantial difference between the two methods, due to the fact that α_t has averaged about 0.6 over the period FY01–FY14 instead of 1.0 (though see the caveats in the note below).[22] As Fig. 4 shows, there are

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[21] The dotted curves in Fig. 4 correspond to the dotted curves in Fig. 1, with the exception of $r{:}C$, which corresponds to Figure 1's solid curve $r{:}\Sigma$, for reasons explained in the previous section.

[22] My adaptation differs from the CSTRS calculations in manners that can be substantive. The differences include the 2–3 year lag between amortization and UAL (due to CSTRS' biennial valuation schedule) and smoothed vs. market valuation. Moreover, the reason $\alpha_t < 1$ for CSTRS differs from the discussion of (7) and (8). Unlike most plans, CSTRS divides its UAL into separate pieces and amortizes them separately. For the main piece, the remaining amortization period (closed) is now short enough to make $\alpha_t > 1$, but there is also a significant piece of *negative* amortization, amortized over a shorter period,

also differences between the two methods in the estimated impact of liability losses, but these are relatively small.

Finally, it is important to note there is *no* difference between the two methods in the estimated impact of contribution shortfalls, $\Delta_{0,T}\text{UAL}^{C:\alpha} = \Delta_{0,T}\text{UAL}^{C:C}$. That is because the counterfactual assumption in both cases is $\text{AMT}_t = r^*\text{UAL}_{t-1}$, such that $C_t = 0$. This does not involve (6) or (8). Thus, when we examine the difference between using exogenous α_t vs. C_t, it will change the *relative* importance of the gap in investment returns and contributions, since the modeling choice affects the measured impact of the former, but not the latter. Under the SS method the impact of the investment gap was about 50% greater than the contribution shortfall for CSTRS, but under my method, it is about 125% greater. This improved method can make a stronger and more accurate impression on policy-makers about the need to cut assumed returns.

7 The impact on cumulative contributions and UAL under exogenous C_t, α_t, and AMT_t

The UAL-drivers also have an impact on cumulative contributions. This is significant, because we are not only interested in payments-yet-to-be made – the UAL – but also the impact on payments that have been made – amortization. I will compare here the impact on cumulative amortization and UAL of the gap in investment returns and liability losses under exogenous C_t and α_t. I will also compare these impacts to the benchmark case of exogenous amortization (formally analyzed in the Appendix), where, by definition, the impact on cumulative amortization is zero. The result is that the total impact is virtually invariant, but the split between paid and unpaid impact varies by assumption.

As illustrated in Fig. 2, the assumed counterfactual behavior of AMT_t depends on what is taken as exogenous: C_t, α_t, or AMT_t itself (depicted there by the horizontal line). For the counterfactual on the gap in investment returns, we have, under exogenous C_t, the expression given in (6), $\text{AMT}_t^{r:C} = r^*\text{UAL}_{t-1}^{r:C} - C_t$, which, in turn implies the difference between the actual and the counterfactual, $\text{AMT}_t - \text{AMT}_t^{r:C} = r^*(\text{UAL}_{t-1} - \text{UAL}_{t-1}^{r:C})$. To put these amortization impacts on a common footing with the UAL impacts, consider their cumulative value, with assumed interest r^*. That is, the asset value of the series of amortization impacts, by year T, is

$$\sum_{t=1}^{T}(1 + r^*)^{T-t}(\text{AMT}_t - \text{AMT}_t^{r:C}) = \sum_{t=1}^{T}(1 + r^*)^{T-t}r^*(\text{UAL}_{t-1} - \text{UAL}_{t-1}^{r:C}).$$

This answers the question, 'How would cumulative amortization differ from the actual if investments had met the assumed return, *and amortization had adjusted dollar-for-dollar to the interest on the resulting UALs*?' It is the counterpart to the 'what if' impact on UAL. It is represented in Fig. 5 as the $7.0 billion blue bar in the first column, atop the $5.0 billion red bar for the UAL impact. The total impact

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and that renders the overall ratio of amortization to interest less than one. Thus, the use here of the overall α_t's "as if" the formula was proportional is solely for illustrative purposes.
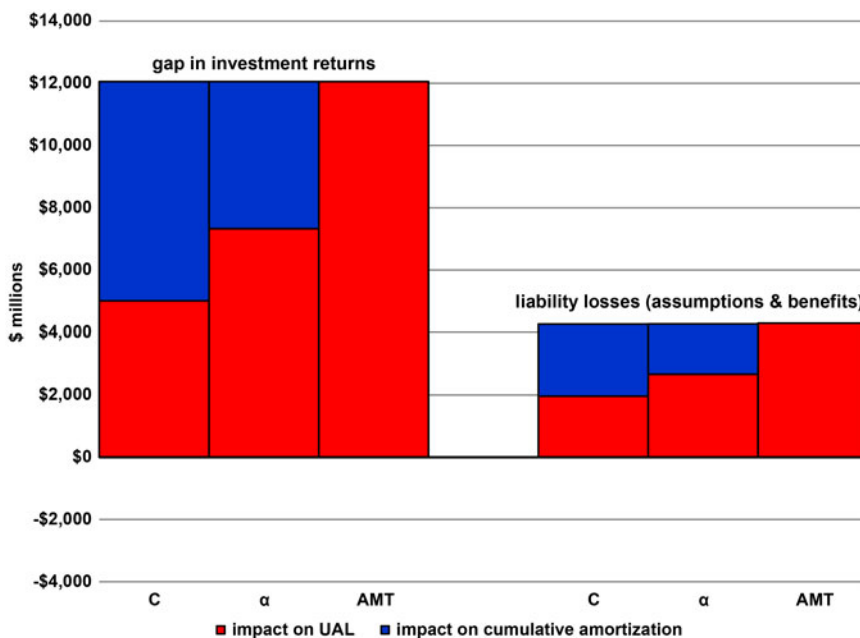
Figure 5. Impact on UAL and cumulative amortization. C_t vs. α_t vs. AMT_t exogenous. Adapted from CSTRS, FY00–FY14.

of the gap in investment returns – UAL plus cumulative amortization – is $12.0 billion under exogenous C_t.

The total impact is the same under exogenous AMT_t. Since there is no amortization impact under exogenous AMT_t, the $12.0 billion red bar for UAL impact in the third column is also the total impact. Thus, the difference between the red bars in the first and third columns of Fig. 5 (the UAL impacts) is equal to the $7.0 billion blue bar in the first column (the impact on cumulative amortization under exogenous C_t).

Further insight can be drawn from the explicit solution to the UAL impact under exogenous AMT_t, provided in the Appendix. As shown there, the UAL impact of the gap in investment returns is the compounded gap between $(1 + r^*)$ and $(1 + r_t)$ over the T-year period, as applied to the initial assets and annual cash flows. Recall that under exogenous C_t the UAL impact is $\sum_{t=1}^{T} (r^* - r_t)A_{t-1}$, i.e., essentially the uncompounded gap. The difference between the two follows from the SS assumption that amortization covers interest on the additional UAL, so there is no compounding. There is, instead, the cumulative impact on amortization.

For my preferred model, exogenous α_t, the total impact is also $12.0 billion, split between $7.3 billion on UAL and $4.7 billion on cumulative amortization, represented by the blue bar in the second column of Fig. 5. In short, the SS method implicitly attributes a substantial portion ($2.3 billion here) of the total impact of the gap in investment returns to amortization instead of UAL. That is, the SS method implicitly assumes that the gap in returns generates much more subsequent amortization than actual formulas would imply, so the UAL impact is that much smaller. More

precisely, the SS counterfactual scenario is that without the gap, amortization would have been \$7.0 billion lower, instead of the \$4.7 billion implied by the amortization formula, so the UAL would have been only \$5.0 billion lower, instead of \$7.3 billion.

Similar observations apply to the impacts of liability losses (including benefit hikes) on the UAL and cumulative amortization. As shown in the Appendix, under exogenous AMT_t, the UAL impact includes the cumulative compound interest $\sum_{t=1}^{T}(1+r^*)^{T-t}(L_t - L_{t|t-1}^e)$, unlike the SS method where no interest accrues, $\sum_{t=1}^{T}(L_t - L_{t|t-1}^e)$. Again, that is because in the SS framework, with exogenous C_t, AMT_t responds to cover the interest on liability losses. With AMT_t exogenous, these losses have no impact on amortization. My preferred model, exogenous α_t, lies in between. Thus, the second triplet in Fig. 5 is similar to the first, but on reduced scale.[23]

One might be tempted to dismiss the impact on cumulative amortization as a matter of little significance since this has already been paid. But this impact is not totally benign. It means that some of the costs have been shifted from the cohort that incurred them to a later (albeit recent) cohort, creating some generational inequity in doing so. In other words, the split of the impact between paid (cumulative amortization) and yet-to-be-paid (UAL) is really a split between generational inequity already imposed and generational inequity to come. It is important to understand both of them. The SS method understates the impact of the gap in investment returns on UAL, by implicitly overstating the impact on cumulative amortization.

8 The liability impact of POBs

The SS method also provides a highly misleading account of POBs' impact on UAL. As noted above, if a state issues POBs to 'pay down,' say, \$2 billion in a fund's UAL (as did CSTRS in 2008) the cumulative impact under the SS method remains at \$2 billion every year thereafter.[24] This is regardless of the returns – positive or negative, above or below r^* – on the proceeds invested in the fund (let alone the outstanding POB debt incurred by the state).[25]

The math behind this is straightforward. Standard actuarial accounting adds the proceeds from any year's bond issue, POB_t, to the fund's assets, A_t, in (3), so the annual gain/loss statement (4) becomes:

$$\Delta_{t-1,t}UAL = (r^* - r_t)A_{t-1} + (r^*UAL_{t-1} - AMT_t) + (L_t - L_{t|t-1}^e)$$
$$- POB_t, \qquad (4\ POB)$$

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[23] For contribution shortfalls, the impact on the UAL is offset by the reverse impact on amortization, since the contribution shortfall *is* (by definition) the reduction in amortization below the interest. Thus, the total impact of the contribution shortfall – paid plus yet-to-be-paid – is essentially zero (not shown in Fig. 5).

[24] The flat impact of the FY08 POB can be read off Table A5 of Aubry and Munnell (2015).

[25] Evaluations of the POB do compare the (actual) returns with the state's POB debt service. The problem is these evaluations are inconsistent with the UAL accounting of the SS method. For example, Aubry and Munnell (2015) provide an internal rate of return evaluation of the CSTRS POB (pp. 51–53), but it is not reconciled with the \$2 billion cumulative impact on UAL reported in Tables 25 and A5 under the SS method.
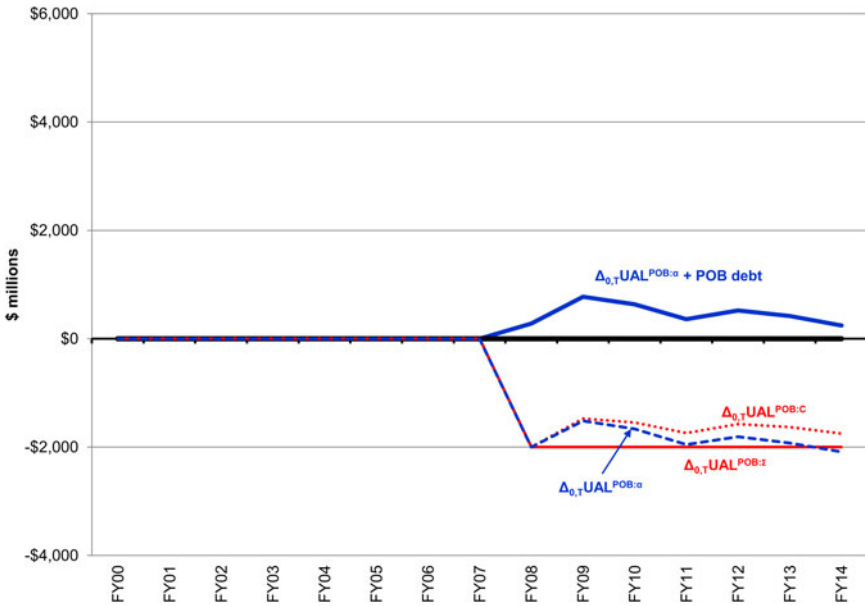
Figure 6. POB impact. SS, exogenous C_t, α_t + POB debt, CSTRS, FY00–FY14 adapted from CSTRS, supplemented by State of Connecticut Annual information statement.

and the SS method's multi-year accounting extends (5) to:

$$
\begin{aligned}
\Delta_{0,T}\text{UAL} &= \sum_{t=1}^{T}(r^*-r_t)A_{t-1} + \sum_{t=1}^{T}(r^*\text{UAL}_{t-1}-\text{AMT}_t) + \sum_{t=1}^{T}(L_t-L^e_{t|t-1}) - \sum_{t=1}^{T}\text{POB}_t \\
&= \Delta_{0,T}\text{UAL}^{r:\Sigma} + \Delta_{0,T}\text{UAL}^{C:\Sigma} + \Delta_{0,T}\text{UAL}^{L:\Sigma} + \Delta_{0,T}\text{UAL}^{\text{POB}:\Sigma}.
\end{aligned}
$$
(5 POB)

Thus, any proceeds from POBs in year t continue unchanged in $\Delta_{0,T}\text{UAL}^{\text{POB}:\Sigma}$ thereafter. Figure 6 shows the FY08 reduction in UAL from that year's POB and the SS method carries it forward as depicted in the flat line beyond FY08, $\Delta_{0,T}\text{UAL}^{\text{POB}:\Sigma}$ (this line was omitted from Fig. 1).

There are three flaws in the method that underlie this result, two of which were discussed above, but not their implications for POB accounting. To see them in this context, it is helpful to note that $\Delta_{0,T}\text{UAL}^{\text{POB}:\Sigma}$ is the difference between the actual rise in UAL, $\Delta_{0,T}\text{UAL}$, as given in the first line of (5 POB) and the SS counterfactual if the POB had not been issued:

$$
\Delta_{0,T}\text{UAL} = \sum_{t=1}^{T}(r^*-r_t)A_{t-1} + \sum_{t=1}^{T}(r^*\text{UAL}_{t-1}-\text{AMT}_t) + \sum_{t=1}^{T}(L_t-L^e_{t|t-1}) - 0,
$$
(5 CF$^{\text{POB}}$)

where the fourth term is zero, since there are no POBs in this counterfactual.

The first flaw is the SS method's exogenous treatment of the series of lagged asset values, A_{t-1}, in the first term above. As discussed earlier, this flaw had relatively minor implications for the other UAL-drivers, but it is more significant for the POB counterfactual. After all, the point of issuing a POB in period t was to add assets, so, in its absence, A_t would be lower – \$2 billion lower in the case of CSTRS' FY08 issue. Consequently, any losses due to subsequent gaps between the actual and assumed returns would be overstated in the SS counterfactual, since those assets would not have been there. The first term of ($5\ \mathrm{CF^{POB}}$) should be lower, and the difference with ($5\ \mathrm{POB}$) – the measured impact – should be greater in algebraic value. To state the result directly, the POB's favorable UAL impact in period t is partially offset in $t + 1$ by the gap in returns on the proceeds, and similarly in following years. This effect is missed by the SS method's treatment of the lagged asset series as exogenous.

CSTRS provides a dramatic example. Immediately following the POB issue of \$2 billion, the fund lost 17.84% in FY09, 26.34 percentage points below the assumed return. Thus, the gap in returns on the \$2 billion was \$527 million that year, offsetting a good portion of the prior year's UAL reduction from the POB proceeds. This can be seen in Fig. 6, on the dotted curve $\Delta_{0,T}\mathrm{UAL^{POB:C}}$, representing the POB's impact on UAL, with endogenous asset values (but still exogenous C). The effect of the FY09 investment loss on the POB's proceeds can be seen by comparing the point on that curve with the solid curve for $\Delta_{0,T}\mathrm{UAL^{POB:\Sigma}}$, lying along the −\$2 billion line. It is certainly intuitive that the \$2 billion reduction in UAL from the FY08 proceeds was attenuated in FY09 by the losses on those proceeds, but it is *not* reflected in the SS method, with exogenous A_{t-1}.

The second flaw underlying the SS method's flat $\Delta_{0,T}\mathrm{UAL^{POB}}$ curve is the implicit amortization assumption that is the main subject of this paper – exogenous C_t, the second term of ($5\ \mathrm{CF^{POB}}$). For exogenous α_t, the $\Delta_{0,T}\mathrm{UAL^{POB:\alpha}}$ curve lies below $\Delta_{0,T}\mathrm{UAL^{POB:C}}$, as shown in the dashed line of Fig. 6. Both curves reflect a cut in amortization payments triggered by the cut in UAL, but they are cut by less on $\Delta_{0,T}\mathrm{UAL^{POB:\alpha}}$ than on $\Delta_{0,T}\mathrm{UAL^{POB:C}}$. Under exogenous C_t, amortization payments are cut by an amount equal to the reduced interest on the UAL, but under exogenous α_t, they are only cut by a fraction of that amount, α_t. The net impact (beyond the initial \$2 billion) under each assumption is the difference between the returns on the proceeds and the reduction in amortization. By FY14, the cumulative return on the proceeds, although less than r^*, was large enough to offset the cut in payments under exogenous α_t, so the net reduction of UAL was greater than the initial \$2 billion proceeds.

The third flaw is that standard UAL accounting omits the POB debt itself. To complete the picture for plans that have relied on POBs, one should integrate the POB debt into the UAL analysis. Although POBs are typically issued by the state, rather than the pension plan, for the taxpayer both the UAL and POB debt are liabilities.[26] It is straightforward to add the outstanding POB debt to the POB impact on UAL.

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[26] Indeed, the full liability is understood in pitching POBs, since they are sold as an arbitrage play, borrowing at one rate and investing in the plan, with a higher assumed return r^*. In the case of CSTRS, the State Treasurer estimated interest at 5.88% over the 25-year life of the bonds, compared with 8.5% assumed return on the proceeds (Denise L. Nappier, as quoted by Munnell *et al.*, 2010). Since the bonds were

The result is depicted in the solid heavy blue line of Fig. 6, $\Delta_{0,T}$UAL$^{\text{POB}:\alpha}$ + POB debt. From the outset, the net liability has been increased, not decreased, by the POB. In addition to borrowing \$2 billion for the CSTRS fund, the first 2 years of interest were also borrowed, as well as the issuing costs, for a total of \$2.277 billion. Thus, for FY08, the net liability rose by \$277 million. The net liability rises further in FY09, due to that year's large investment losses, reflected in $\Delta_{0,T}$UAL$^{\text{POB}:\alpha}$. In addition, the debt service is highly back-loaded, so the outstanding POB debt continues to rise (widening the gap between the heavy blue line and the dashed line).[27] The net liability has drifted down since FY09, due to generally good investment returns, but remains in positive territory as of FY14: the POB debt of \$2.333 billion outweighs my estimate of the UAL impact, $-\$2.088$ billion.[28]

9 Additivity vs. interactions

A distinctive feature of the SS method is its additivity. As noted earlier, equation (5) (and its POB version, (5 POB)) provide a decomposition that adds up to the actual change in UAL. The alternative proposed here does not. Indeed, the first correction – endogenizing asset values – breaks additivity, even before replacing the amortization assumption. This can be seen in Fig. 1, where the sum of the dotted curves deviate (albeit only slightly) from the solid curves (which do add up), and when POBs are added in Fig. 6, the deviation is a bit greater.

The proposed alternative amortization assumption disrupts additivity more substantially. As Figs 4 and 5 show, adding up the UAL impacts under exogenous α_t can well exceed the sum under exogenous C_t (and the similar SS estimates) and, hence, the actual rise in UAL. As mentioned earlier, the reason is that there are large interaction effects, most notably between the gap in investment returns and the amortization formula ($\alpha_t < 1$). These interactions are intertemporal, rather than contemporaneous, which is why the multi-year decomposition is problematic, even if the annual gain/loss statement is not.[29]

What are we to make of the adding-up problem? 'Not too much,' in my view. The goal of a policy-relevant exercise is to evaluate useful counterfactuals, not to parcel out the rise in UAL in a fashion that adds up neatly. Indeed, if the solution adds

issued in FY08, I estimate the return on CSTRS was 6.37% through FY14, well below 8.5%, but above 5.88%.

[27] The yearly outstanding POB balance is reported in State of Connecticut (2014), p. II-9. It will continue to rise until FY22. The bond issue was complex – 15 current interest bonds (no principal paid until maturity) and six capital appreciation bonds (no principal or interest paid until maturity) of different maturities and rates. The annual debt service payments from the 21 bonds is not disclosed, but can be computed from information in State of Connecticut, 2008. They are fairly flat around \$125 million through FY21, but then jump to over \$300 million by FY23.

[28] Adding in the cumulative value of amortization payments saved and POB debt service paid makes the result for the total impact of the POB (analogous to the total impacts in Fig. 5) a bit more positive (i.e., more adverse).

[29] By way of analogy, for $\Delta(xy) = (x_1y_1) - (x_0y_0)$, using the (higher) period 1 values as our base, we attribute $y_1\Delta x$ to Δx and $x_1\Delta y$ to Δy, which sum to $\Delta(xy) + \Delta x\Delta y > \Delta(xy)$, and conversely if we use the (lower) period 0 values as our base. The 'what-if' in the UAL analysis corresponds to the former, using the actual values (drivers that generate higher UAL) as the base and counterfactual values (drivers that generate no change in UAL) for single-driver attribution. That is why the sum of the UAL impact of individual drivers will exceed the actual rise in the UAL.

up for faulty counterfactuals – often misunderstood – we misinform the user. If the user's interest is the impact of the gap in investment returns, we should use the right counterfactual to answer that question. That is, the policy-relevant question is 'what has been the impact on UAL of the gap in investment returns, given the *actual* amortization formula?' rather than the artificial version answered by the SS method: 'what would have been the impact on UAL of the gap in investment returns if the amortization formula had covered interest?' Additivity may be a short-hand way of indicating relative importance (*X* percent due to one factor and *Y* percent due to another), but if the price is assuming away important interactions, the effort is misguided. The relative importance of the UAL-drivers can be readily calculated from their individual impacts, including interaction effects. As noted earlier, CSTRS' UAL impact from the gap in returns is about 50% greater than that of the contribution shortfall under exogenous C_t, but under exogenous α_t, it is about 125% greater – a big difference.

10 Conclusion

The proponents of the SS method characterize it as a 'new tool' that presents a 'clean story...to cut through the political rhetoric and identify why a plan is in trouble' (Munnell *et al.*, 2015, p. 1). They advocate adding it to every plan's annual actuarial valuation; and one state has commissioned a study based on this method. In this paper, I have argued for backing up to carefully specify the questions we want to answer and the counterfactuals that go with them, and have found the SS method wanting.

One of the biggest questions is the impact of the gap between assumed and actual investment returns. As investment analysts and others conclude that the likely returns going forward are lower than long-term historic norms (Dobbs *et al.*, 2016) and as pension boards appear reluctant to reduce their assumptions accordingly, the UAL analysis has profound implications for the policy decision on the assumed return.[30] That decision can be swayed by the relative importance of the gap in returns for the rise of the UAL over the past 15 years.

The SS method answers the specific question, 'What would be the impact on UAL if investments had met the assumed return, *and amortization had adjusted dollar-for-dollar to the interest on the resulting UAL*?' In defense of this method, one might argue that it isolates the direct impact of the shortfall in returns, leaving aside any indirect impact through the amortization formula's incomplete adjustment to the interest on additional UAL. However, it seems unlikely that users of this method understand this nuance, since it has not been previously identified. More likely, users believe the method answers the question, 'What would be the impact on UAL if investments had met the assumed return, *and amortization had adjusted according to the actual policy*?' I have shown that the answer to this question can be considerably larger than the one addressed by the SS method.

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[30] Even if the expected return is realistic, virtually the entire finance economics profession (led by Brown and Wilcox (2009) and Novy-Marx and Rauh (2009, 2011)) has argued against using it to discount pension liabilities, since those liabilities are largely risk-free and therefore more like bonds.

The other main policy implication of this analysis is the importance of revising the amortization formula to make sure it at least covers interest on the UAL. That is the main policy thrust of the SS method (Munnell *et al.*, 2015, p. 5), through its measure of the impact of contribution shortfalls, and it is a valuable point. In this paper, I have spelled out the specific combinations of amortization parameters – long amortization periods and high assumed payroll growth – that create the shortfall. These are policy levers under control of the pension fund, and it is important that they be chosen with a full public understanding of their impact.

However, a high assumed rate of return is another policy lever that contributes to the shortfall between amortization and interest, since the assumed return is the assumed interest on liabilities. That is, the assumed return factors directly into the UAL impact of both the gap in returns *and* the shortfall in contributions (as well as the interaction between the two). Thus, a full understanding of the need to cut the assumed return rests on a complete and realistic analysis of its impact on the UAL.

It is, therefore, important to be clear that although my proposed method here improves on the SS method, neither of these methods *directly* address the impact of high assumed returns. It is the actual return that varies in the counterfactual ('what if investments had met the assumed return'), not the assumed return. That follows from the fact that liabilities are held constant in the counterfactual on investment returns, which would not be true for any counterfactual on the assumed return. As a result, both of these methods might lead the user to conclude that the UAL impact of the gap is simply a measure of the fund's bad luck, rather than a policy decision to assume high returns. My method gives a more accurate answer to the impact of failing to meet assumed returns and may, therefore, provide a greater impetus to cut the assumed return, but it does not actually assess the counterfactual impact of such a cut. It is a different question to ask, 'What would be the impact on UAL if the assumed return had been set to the actual average return?' The counterfactual for this question would adjust liabilities (as in the finance economics literature cited above) and further adjust contributions, via the UAL and amortization, as well as normal cost. To the extent that historical experience can help guide policy on how important it is to revise $r*$ (and how much), this would be a more relevant exercise, but that is a topic for further research.[31] In the meantime, since the currently influential literature assesses the UAL impact of the failure to meet assumed returns, it is important to get the right answer to that question, to at least indirectly inform the policy decisions at hand.

References

Aubry, J-P. and Munnell, A. H. (2015) Final report on Connecticut's state employees retirement system and teachers' retirement system. Center for Retirement Research at Boston College, November 2015.

Biggs, A. G. (2015) The state of public pension funding: are government employee plans back on track? American Enterprise Institute, Economic Perspectives, September 2015.

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[31] See Costrell (2016) for an analysis of the steady-state impact of high assumed returns.

Brown, J. and Wilcox, D. (2009) Discounting state and local pension liabilities. *American Economic Review*, **99**(2): 538–542.

Connecticut State Teachers' Retirement System, various years, Actuarial Valuation.

Costrell, R. M. (2016) The 80 percent pension funding target, high assumed returns, and generational inequity. *Contemporary Economic Policy*, forthcoming, special issue on aging and pensions.

Dobbs, R., Koller, T., Lund, S., Ramaswamy, S., Harris, J., Krishnan, M. and Kauffman, D. (2016) Why Investors May Need to Lower their Sights. McKinsey Global Institute, Brussels, San Francisco, and Shanghai.

Malloy, D. (Governor of Connecticut) (2015) Connecticut's Economic and Budgetary Reality, PowerPoint presentation, Hartford, CT, 28 October 2015.

Munnell, A. H., Calabrese, T., Monk, A. and Aubry, J-P. (2010) Pension Obligation Bonds: Financial Crisis Exposes Risks, Center for Retirement Research at Boston College, Number 9, January 2010.

Munnell, A. H., Aubry, J-P. and Cafarelli, M. (2015) How Did State/Local Plans Become Underfunded? Center for Retirement Research at Boston College, Number 42, January 2015.

Novy-Marx, R. and Rauh, J. D. (2009) The liabilities and risks of state-sponsored pension plans. *Journal of Economic Perspectives* **23**(4): 191–210.

Novy-Marx, R. and Rauh, J. D. (2011) Public pension promises: how big are they and what are they worth? *Journal of Finance* **66**(4): 1211–1249.

State of Connecticut (2008) New Issue. $2,276,578,270.55 State of Connecticut, Taxable General Obligation Bonds, 16 April 2008.

State of Connecticut (2014) Annual Information Statement, February 28, 2014, revised 8 December 2014.

Appendix: Explicit Solution for Exogenous AMT_t

In this Appendix, I consider exogenous AMT_t. In the context of (6) and (8), this is the extreme counterfactual where AMT_t does not vary at all with the interest on counterfactual changes in UAL. In Fig. 2, we are rotating the line all the way around the actual values of AMT_t and r^*UAL_{t-1}, from (6) to (8) to the actual AMT_t line itself, where the slope is zero. I examine this extreme case in detail because it generates clear analytical results that may serve to clarify the intermediate case (8), discussed in the text, where amortization responds to changes in the UAL, but by less than the interest on the UAL. It may also be relevant to the case where budgeters do not fund the amortization formula, but simply contribute 'what they can afford.'

I derive the reduced form expression for $\Delta_{0,T}UAL$ by solving for the series A_t and UAL_t in terms of the exogenous series: $(L_t - L^e_{t|t-1})$, r_t, AMT_t, $(NC–B)_t$, POB_t, and the initial conditions A_0 and UAL_0. Begin with the system (3) – (4) in matrix form (including any POBs):

$$\begin{pmatrix} A \\ UAL \end{pmatrix}_t = M_t \begin{pmatrix} A \\ UAL \end{pmatrix}_{t-1} + v_t,$$

where

$$M_t = \begin{bmatrix} (1+r_t) & 0 \\ (r^*-r_t) & (1+r^*) \end{bmatrix} \text{ and } v_t = \begin{bmatrix} AMT + (NC - B) + POB \\ (L - L^e) - AMT - POB) \end{bmatrix}_t.$$

This implies

$$\begin{pmatrix} A \\ \text{UAL} \end{pmatrix}_T = \boldsymbol{M}_T \boldsymbol{M}_{T-1} \dots \boldsymbol{M}_1 \begin{pmatrix} A \\ \text{UAL} \end{pmatrix}_0 + \boldsymbol{M}_T \boldsymbol{M}_{T-1} \dots \boldsymbol{M}_2 v_1 + \dots + \boldsymbol{M}_T v_{T-1} + v_T.$$

It can be shown, by induction on $\boldsymbol{M}_T \boldsymbol{M}_{T-1} \dots \boldsymbol{M}_{t+1}$ that

$$\begin{pmatrix} A \\ \text{UAL} \end{pmatrix}_T = \begin{bmatrix} \prod_{t=1}^{T}(1+r_t) & 0 \\ \left[(1+r^*)^T - \prod_{t=1}^{T}(1+r_t)\right] & (1+r^*)^T \end{bmatrix} \begin{pmatrix} A \\ \text{UAL} \end{pmatrix}_0$$
$$+ \left(\sum_{t=1}^{T-1} \begin{bmatrix} \prod_{\tau=t+1}^{T}(1+r_\tau) & 0 \\ \left[(1+r^*)^{T-t} - \prod_{\tau=t+1}^{T}(1+r_\tau)\right] & (1+r^*)^{T-t} \end{bmatrix} v_t \right) + v_T.$$

From this expression we have:

$$\Delta_{0,T}\text{UAL} = \left[(1+r^*)^T - \prod_{t=1}^{T}(1+r_t)\right]A_0 + \left[(1+r^*)^T - 1\right]\text{UAL}_0$$
$$+ \sum_{t=1}^{T-1}\left[(1+r^*)^{T-t} - \prod_{\tau=t+1}^{T}(1+r_\tau)\right]\left[\text{AMT}_t + (\text{NC}-B)_t + \text{POB}_t\right]$$
$$+ \sum_{t=1}^{T}(1+r^*)^{T-t}\left[(L_t - L_{t|t-1}^e) - \text{AMT}_t - \text{POB}_t\right].$$

(A1)

For the actual values of all the variables, (A1) gives the same result as (5 POB). The counterfactuals, however, will take AMT_t as exogenous instead of C_t or α_t.

One can readily find the UAL impact from (A1) for liability losses and POBs. For liability losses, $(L_t - L_{t|t-1}^e)$, simply take the difference between the actual $\Delta_{0,T}\text{UAL}$ given in (A1) and the counterfactual value of (A1) with $(L_t - L_{t|t-1}^e) = 0$. Since everything else in (A1) is unchanged (exogenous or pre-determined), the result is straightforward:

$$\Delta_{0,T}\text{UAL}^{\text{L:AMT}} = \sum_{t=1}^{T}(1+r^*)^{T-t}\left(L_t - L_{t|t-1}^e\right),$$

where the superscript AMT denotes its exogeneity in the counterfactual. Here, the UAL impact of liability losses includes the cumulative interest (at assumed rate r^*), unlike the SS method where no interest accrues, $\Delta_{0,T}\text{UAL}^{L:\Sigma} = \sum_{t=1}^{T}(L_t - L_{t|t-1}^e)$, as discussed in the text.

Similarly, it is straightforward to show that the reduction in the UAL due to the POB will include the investment return, at the actual rate, r_t. The difference between (A1) and its counterfactual version with $\text{POB}_t = 0$ simplifies to:

$$\Delta_{0,T}\text{UAL}^{\text{POB:AMT}} = -\sum_{t=1}^{T-1}\left[\prod_{\tau=t+1}^{T}(1+r_\tau)\right]\text{POB}_t - \text{POB}_T.$$

I turn now to our main focus, the impact of the gap in investment returns. The counterfactual value of $\Delta_{0,T}\text{UAL}$, is found by setting $r_t = r^*$ in (A1). This zeros out the first and third terms and leaves the other terms unchanged, under exogenous AMT_t. Thus, the

difference between the actual $\Delta_{0,T}\text{UAL}$ and its counterfactual value is the first and third terms:

$$\Delta_{0,T}\text{UAL}^{r:\text{AMT}} = \left[(1+r^*)^T - \prod_{t=1}^{T}(1+r_t)\right]A_0$$

$$+ \sum_{t=1}^{T-1}\left[(1+r^*)^{T-t} - \prod_{\tau=t+1}^{T}(1+r_\tau)\right][\text{AMT}_t + (\text{NC}-B)_t + \text{POB}_t].$$

This UAL impact is simply the compounded difference between $(1+r^*)$ and $(1+r_t)$ over the T-year period, as applied to the initial assets and the annual cash flows (all of which are exogenous at the actual levels).[32]

Equation (A1) illuminates the difference between the UAL impact of the gap in investment returns under exogenous C_t and AMT_t. The difference is in the counterfactual value of AMT_t in the last term of (A1). Under exogenous C_t, that value is not the same as the actual value of AMT_t, so it does not cancel out when taking the difference. The counterfactual for AMT_t is instead given by (6), $\text{AMT}_t^{r:C} = r^*\text{UAL}_{t-1}^{r:C} - C_t$, where the superscripts denote the counterfactual values, under exogenous C_t, as distinct from the actual values. Thus, $\Delta_{0,T}\text{UAL}^{r:C}$ is the difference between the actual value of (A1) and the counterfactual value of (A1) with $r_t = r^*$ and $\text{AMT}_t^{r:C}$:[33]

$$\Delta_{0,T}\text{UAL}^{r:C} = \left[(1+r^*)^T - \prod_{t=1}^{T}(1+r_t)\right]A_0$$

$$+ \sum_{t=1}^{T-1}\left[(1+r^*)^{T-t} - \prod_{\tau=t+1}^{T}(1+r_\tau)\right][\text{AMT}_t + (\text{NC}-B)_t + \text{POB}_t]$$

$$+ \sum_{t=1}^{T}(1+r^*)^{T-t}[\text{AMT}_t^{r:C} - \text{AMT}_t].$$

(A2)

The difference between $\Delta_{0,T}\text{UAL}^{r:C}$ and $\Delta_{0,T}\text{UAL}^{r:\text{AMT}}$ is the third term. This term is negative, since the counterfactual values of AMT_t are reduced under exogenous C_t when $r_t = r^*$ reduces the UALs. Specifically, $\Delta_{0,T}\text{UAL}^{r:\text{AMT}}$ exceeds $\Delta_{0,T}\text{UAL}^{r:C}$ by $\sum_{t=1}^{T}(1+r^*)^{T-t}(\text{AMT}_t - \text{AMT}_t^{r:C})$. Substituting for $\text{AMT}_t^{r:C}$ from above, and using the definition of C_t, we find that $\Delta_{0,T}\text{UAL}^{r:\text{AMT}}$ exceeds $\Delta_{0,T}\text{UAL}^{r:C}$ by $\sum_{t=1}^{T}(1+r^*)^{T-t}r^*(\text{UAL}_{t-1} - \text{UAL}_{t-1}^{r:C})$. As discussed in the text, this equals the cumulative value (with interest) of the reduction in amortization, under exogenous C_t, that would have matched the interest on the reduction in UAL, had investment met returns r^*. Under exogenous AMT_t these payments would not have been reduced, so the full impact of the gap in investment returns would fall on UAL, instead of being offset by amortization, as under the SS method. This is also the difference between the gap in compounded returns given in $\Delta_{0,T}\text{UAL}^{r:\text{AMT}}$ above and the gap in uncompounded returns in the SS method, $\Delta_{0,T}\text{UAL}^{r:C} = \Delta_{0,T}\text{UAL}^{r:\Sigma} = \sum_{t=1}^{T}(r^* - r_t)A_{t-1}$.

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[32] There is an additional term for the cash flows for period T. This is not affected by r_T, so it is the same under the actual and the counterfactual, and does not enter the impact of the gap in investment returns.

[33] The result equals (5)'s $\sum_{t=1}^{T}(r^* - r_t)A_{t-1}$, but the derivation from (A1) is more readily compared with $\Delta_{0,T}\text{UAL}^{r:\text{AMT}}$.