CONSISTENT CONJECTURES IN MONOPOLISTIC COMPETITION

Robert M. COSTRELL

University of Massachusetts, Amherst, MA 01003, USA

Final version received March 1989

Models of monopolistic competition, such as Dixit and Stiglitz, and Spence, typically assume firms take market variables as parametric. This conjectural variation corresponds to Chamberlin’s 'large group' approximation, although these models need not have a large number of firms. The present note, following Perry, shows that consistent conjectures about the size and number of rivals can provide a more satisfactory basis for this assumption.

1. Introduction

What is an appropriate conjectural variation to use in models of product differentiation? Two immediate candidates are the Nash conjectures with respect to price or quantity. However, Dixit and Stiglitz (1977), Spence (1976, Section 5), and other papers on monopolistic competition assume a conjecture in which firms take the relevant summary statistic of industry outputs as independent of their own actions. In the limiting case of homogeneous products, this is the competitive conjecture; and we follow other papers in extending this terminology to models of product differentiation.1

Chamberlin (1962) rationalizes the competitive conjecture by appeal to 'large group' considerations.2 However, the 'large group' approximation is not an appropriate basis for the competitive conjecture when used by Dixit and Stiglitz (1977), and Spence (1976), since their models need not have a large number of firms. Indeed, the main point of their models is to argue that the equilibrium number of firms is too small, relative to the optimum.

The present note provides an alternative rationale for the competitive

---

1 I would like to acknowledge the helpful comments of seminar participants at Brandeis University, and a particularly conscientious referee of this journal.
2 See Koenker and Perry (1981) for a discussion of the implications for product diversity when non-competitive conjectures are used in Spence's model of product differentiation. Ireland (1983) extends the analysis to a model which allows firms to choose the number of product varieties.
3 When the number of firms grows large, as in Hart (1979, 1985a, b), the Nash conjectures in price or quantity converge with each other, and with the competitive conjecture. Hart examines some conditions under which the firm's demand curve becomes perfectly elastic or retains a negative slope as the number of firms becomes infinite. Wolinsky (1986) considers imperfect information in this context.
conjecture, by showing that under free entry it is 'consistent'. Although most of the consistent conjectures literature takes the number of firms as given, Perry (1982, Section 4) points out that to be 'fully' consistent, a firm should form conjectures about the effect of its actions on the number, as well as the size, of its rivals.\(^3\) For the case of homogeneous products, he shows that the recognition of free entry results in the consistent conjecture being the competitive conjecture, whether or not the number of firms is large. In other words, the consistent conjecture is for firms to take market output, and therefore price, as given. The present note extends his analysis to the symmetric differentiated case and shows that under free entry, the firm acts as if the relevant market variable is independent of its actions. As a result, the firm’s perceived demand curve is more elastic than with the Nash assumption on price or quantity.

The note is organized as follows: section 2 specifies the technology and preferences. Section 3 extends Perry’s result on fully consistent conjectures to the symmetric differentiated case. Section 4 uses slightly stricter assumptions to derive further results. Concluding remarks discuss extensions to the model, treated in related papers.

2. Technology and preferences

Following Dixit and Stiglitz (1977) and Spence (1976) we model the benefits of variety by assuming that all consumers are alike,\(^4\) but have strictly convex preferences over a very large number of potential differentiated outputs, \(\tilde{n}\), of which only \(n\) will actually be produced. For simplicity, we assume preferences take the following separable form:

\[
U = U\left[ S = \sum_{i=1}^{n} g(y_i), L = \sum_{i=1}^{n} g(y_i), g' > 0, g'' < 0. \right. \]

(1)

Here, \(L\) is the good from the undifferentiated sector (e.g. leisure, or, if labor is supplied inelastically, \(L\) can be some homogeneous output produced under constant returns to labor). The maximum amount of the undifferentiated good available is \(\tilde{L}\), and \(X\) of it is used in the differentiated sector. The \(y\)'s are the symmetrically differentiated products, and \(S\) is a summary statistic for them. This is a slight generalization of the CES case examined by Dixit and

\(^3\)See Bresnahan (1981, 1983), Robson (1983), and Kamien and Schwartz (1983). See also Ulph (1983), who examines consistent conjectures in duopoly models where only one firm may be active.

\(^4\)Hart (1979, 1985a, b) models the benefits of variety at the aggregate level by assuming that tastes differ, but are known to arise from a given distribution.
Stiglitz (1977, Section 1), $g = y^\rho$ (the degenerate case $\rho = 1$ is homogeneous products). The representative consumer maximizes (1) subject to

$$\sum_{i=1}^{n} p_i y_i = X + \sum_{i=1}^{n} \pi_i, \tag{2}$$

where $\pi_i$ is the profit of firm $i$, and $L$ is numéraire. Consumer equilibrium gives us the price of good $i$, in terms of $L$:

$$p_i = (U_S/U_L)g_i = p(S, X, y_i, p_S, p_X, p_{y_S}, < 0, \tag{3}$$

where subscripts, other than $i$, denote partial derivatives.

The technology of producing the differentiated products is characterized by the cost function, in terms of foregone output from the undifferentiated sector (e.g. leisure):

$$x_i = c(y_i), \ i = 1, \ldots, n, \ c' > 0. \tag{4}$$

We suppose there is a non-convexity in production, sufficiently pronounced that only $n < n$ goods will be produced in equilibrium. We restrict our attention to symmetric equilibrium among these $n$ goods.

3. Consistent conjectures equilibrium

In this section we examine the symmetric equilibrium with consistent conjectures, and generalize Perry's analysis of free entry in two ways. First, we extend his result to the case of differentiated products. Second, we consider not only the conjecture on the summary statistic $S$ of the differentiated sector's outputs, but also the conjecture on its inputs $X$, and hence, the undifferentiated sector's output. In general, both affect the firm's price.

We find that although neither conjecture is generally zero, the relevant weighted sum of the conjectures is zero. As a result, firms with consistent conjectures behave "as if" their actions have no effect on either the index $S$ of

---

5This is a non-Walrasian consumer, since the summations go to $n$, rather than $\bar{n}$. That is, consumers are rationed (to zero) on the $(\bar{n} - n)$ inactive markets, as pointed out by Hart (1979, 1985a,b). Without such rationing, equilibrium does not exist.

6Spence (1976) and others assume that preferences are linear in $L$, such that $X$ does not appear in (3) and (5), below, in which case the analysis is particularly simple. Section 4 will consider the assumption that preferences over $S$ and $L$ are homothetic.

7For example, the non-convexity may be in the form of quasi-fixed costs, such that $c(0) = 0$, but $\lim_{y \to 0} \sigma(y) > 0$. 
the differentiated sector's outputs or on its inputs $X$. Thus, even without a large number of firms, competitive behavior arises by imposing consistency on conjectures in which each firm recognizes the effect of its actions on the number of firms.

We begin by specifying symmetric equilibrium, contingent upon the conjectures. Firm $i$'s profit function can be written as

$$\pi_i = \pi(S, X, y_i) = p(S, X, y_i) - c(y_i).$$

(5)

Taking $y_i$ as the decision variable, profit-maximization gives us

$$\lambda \pi_S(S, X, y_i) + \mu \pi_X(S, X, y_i) + \pi_y(S, X, y_i) = 0,$$

(6)

where subscripts on $\pi$ denote partial derivatives, and the conjectures are

$$\lambda = dS/dy_i, \quad \mu = dX/dy_i.$$

(7)

While it will not in general be true that the consistent conjectures $\lambda = \mu = 0$, we will show that $\lambda \pi_S + \mu \pi_X = 0$ in equilibrium, so the profit-maximizing firm need only consider the partial derivative $\pi_y$, choosing its output $y_i$ such that $\pi_y = 0$.

Specification of symmetric equilibrium is completed by the following conditions:

$$\pi(S, X, y_i) = 0 \text{ (the free entry condition)},$$

(8)

$$S = ng(y_i),$$

(9)

$$X = nc(y_i).$$

(10)

Assuming a solution exists, the system (6), (8)–(10) yields $S$, $X$, $y_i$, and $n$, conditional on the conjectures $\lambda$ and $\mu$.

To find consistent conjectures $\lambda$ and $\mu$ around the symmetric equilibrium, let firm $j$ consider the effect of varying its output on the rest of the system. Conditions (6) and (8) remain as before, but (9) and (10) are modified to

$$S = (n - 1)g(y_i) + g(y_j),$$

(9')

$$X = (n - 1)c(y_i) + c(y_j).$$

(10')

It is not necessary to assume that $\lambda$ and $\mu$ are constants for our main result, presented in this section. In the next section, that assumption will be made to simplify other results.

We ignore the integer problem.
The system (6), (8), (9'), (10') yields the industry equilibrium \( S, X, y, \) and \( n \), contingent on the output of firm \( j \), \( y_j \), and the conjectures \( \lambda \) and \( \mu \). Comparative statics with respect to \( y_j \) gives us consistent conjectures for \( S \) and \( X \) (i.e. \( \lambda \) and \( \mu \) are given in implicit form), as well as consistent conjectures for \( y \) and \( n \).

Since we are not immediately interested in the conjecture for \( n \), it will simplify the presentation to remove it from the system for the moment. Hence, the information in (9')-(10') can be condensed to

\[
[S-g(y_j)]c(y_j) = [X-c(y_j)]g(y_j).
\]

The comparative statics of the system (6), (8), and (11) are then given in matrix form by \( J[dS\,dX\,dy_j] = [0\,0\,b']dy_j \), where

\[
J = \begin{bmatrix}
J_{11} & J_{12} & J_{13} \\
\pi_S & \pi_X & \pi_y \\
\epsilon_i & -g_i & J_{33}
\end{bmatrix}, \quad b = \epsilon g_j - g_i \epsilon_j.
\]

In general, \( b \neq 0 \), so variations in \( y_j \) have a non-zero effect on \( S \) and \( X \). Thus, the separate consistent conjectures do not vanish, i.e. \( \lambda, \mu \neq 0 \). However, from (6), it is the weighted sum \( \lambda \pi_S + \mu \pi_X = \pi_S \Sigma(S/dy_j) + \pi_X (dX/dy_j) \) that is relevant for the firm’s output choice. Using Cramer’s rule, we have:

\[
(\lambda \pi_S + \mu \pi_X) = -b \pi_y |J_{33}|/|J|,
\]

where \( |J_{33}| \) is \( J_{33} \)'s minor. Substituting from (12) into the first-order condition (6), we see that

\[
(\lambda \pi_S + \mu \pi_X) = \pi_{yi} = 0,
\]

provided \( |J| - b|J_{33}| \) does not vanish.\(^{12}\)

This establishes our claim that under consistent conjectures firms act ‘as if’ their actions have no effect on \( S \) and \( X \). This means that each firm need only consider the partial derivative of its inverse demand function, \( p(S, X, y_i) \), with

\(^{10}\)It can be shown that \( b = 0 \) in equilibrium for the CES case, and hence, \( \lambda = \mu = 0 \). Also, in the case of quasilinear preferences, \( \pi_x \equiv 0 \), so, by (13) below, \( \lambda = 0 \), and \( \mu \) is irrelevant. The present analysis can also be applied to Ireland’s (1983) model, where firms choose their output and the number of product varieties. Here, too, consistent conjectures regarding the effect of the two choice variables on the relevant market index are each zero, as a consequence of the assumption of quasilinearity.

\(^{11}\)In general \( |J| \) will not vanish, as will be shown under the stricter assumptions of section 4.

\(^{12}\)Under the stricter assumptions of section 4, \( |J_{33}| = 0 \), and \( |J| < 0 \), so the condition is satisfied.
respect to its output, $y_j$. Thus, the Chamberlinian 'large group' assumption is consistent irrespective of the number of firms. It should be noted that in the case of homogeneous output, $y_j$ does not enter the inverse demand function independently of $S = \Sigma y_i$, so the price-taking conjecture is consistent, as Perry showed.

4. Further results

In this section, we briefly consider further results, based on slightly stronger assumptions. Specifically, take $\lambda$ and $\mu$ as constant, at least in the neighborhood of equilibrium,\(^{13}\) and also assume that preferences are homothetic over $S$ and $L$, as in Dixit and Stiglitz (1977, Section 2).

The assumption that $\lambda$ and $\mu$ are constant allows us to find $J_{13} = \pi_{y_{y_j}}$.

The further assumption of homothetic preferences can be used to show that $|J_{33}| = 0$.\(^{14}\) Since $J_{23} = \pi_{y_S} = 0$ in equilibrium,

\[
|J| = -\pi_{y_{y_j}}[g(y_j)\pi_S + c(y_j)\pi_X] < 0,
\]

since $\pi_S, \pi_X, \pi_{y_{y_j}} < 0$. These results allow us to show that

\[
\lambda = b\pi_X/(g_i\pi_S + c_i\pi_X) \quad \text{and} \quad \mu = -b\pi_S/(g_i\pi_S + c_i\pi_X),
\]

such that $\text{sign}(\lambda) = \text{sign}(b)$ and $\text{sign}(\mu) = -\text{sign}(b)$. This is more readily interpreted by noting that in equilibrium it can be shown that $b$ has the opposite sign of the derivative of $g$'s elasticity with respect to $y$.\(^{16}\)

More importantly, under these assumptions, we have the consistent conjecture

\[
\frac{dy_j}{dy_j} = b|J_{33}|/|J| = 0.
\]

Finally, it can be shown that the consistent conjecture\(^{17}\)

\[
\frac{d\pi}{dy_j} = -\frac{\pi_S g_i + \pi_X c_i}{\pi_S g_j + \pi_X c_j} < 0.
\]

\(^{13}\) Ulph (1983) provides an axiomatic basis for this assumption.

\(^{14}\) We have $J_{13} = \lambda \pi_{x_S} + \mu \pi_{x_j} + \pi_{y_{y_j}}$. Using the previous section's result that $\pi_{y_{y_j}} = 0$ in equilibrium, Cramer's rule gives us $\lambda = -b\pi_X J_{13}/|J|$ and $\mu = \pi_S J_{13}/|J|$. Substituting into $J_{13}$ and using (3) and (5) to show that $(\pi_S \pi_{y_j} - \pi_X \pi_{x_j}) = 0$, we find $J_{13} = \pi_{y_{y_j}}$.

\(^{15}\) Homotheticity gives us $\pi_S \pi_{x_S} + \pi_X \pi_{x_X} - 2\pi_S \pi_{x_S} \pi_{x_X} = 0$, which, along with results from note 14, is used to show $|J_{13}| = 0$.

\(^{16}\) Thus $b > 0$ in the CES case, as noted earlier; $b < 0$ for the example given in Dixit and Stiglitz (1977, section 2), $g(y) = (k + my)^{\nu}$; and $b > 0$ for the modification proposed by Pettengill (1979), $g(y) = (k + my)^{\nu} - k^n$.

\(^{17}\) Differentiate (9') and (10') with respect to $y_j$, yielding expressions for $\lambda$ and $\mu$; use (16); substitute into (13) and rearrange.
Hence, the reason the firm acts ‘as if’ its actions have no effect on \( S \) and \( X \) is that its actions will be offset by entry or exit.\(^{18}\)

5. Conclusion

This note has examined consistent conjectures for firms producing symmetrically differentiated products, while recognizing the effect of their actions on the number and size of their rivals. The main result is that firms act ‘as if’ they have no effect on the relevant market variables. This is the competitive assumption, as generalized to the case of differentiated products, an assumption which has been widely adopted since Dixit and Stiglitz (1977) and Spence (1976). Our result provides a better foundation for that assumption.

The significance of adopting the competitive conjecture, rather than the Nash conjecture in price or quantity, is that the firm’s perceived demand curve is more elastic than under either of the Nash conjectures.\(^{19}\) Consequently, the markup over marginal cost is smaller.

The main result can be extended to cases where there are several relevant market variables, and these variables need not be restricted to the additive form used in this note. For example, a related article [Costrell (1986a)] applies this result to the mean–variance model, where the mean and the variance of the outputs from the risky sector are the relevant summary statistics. The results can also be generalized to the case where there are several inputs, including semi-public inputs, such as R & D, as in Costrell (1986b). Here, the consistent conjecture takes the market’s total R & D as given, rather than the R & D of one’s rivals, i.e. the Nash assumption in inputs adopted by Ruff (1969) and Spence (1984) in their analysis of spillovers. As a result, the free-rider problem is somewhat more serious.

\(^{18}\)This mechanism was discussed, in the homogeneous case, by Fama and Laffer (1972).

\(^{19}\)Specifically, it can be shown that the slopes of the demand curve under the consistent conjecture, the Nash conjecture in prices, and the Nash conjecture in quantities are, respectively,

\[ p_x > p_x + p_x (p_x - p_x' - p_x'') /[p_x + (n - 1)(p_x - p_x'')] > p_x + p_x' + p_x''. \]

References


Costrell, R.M., 1986b, Appropriability, duplication, and diversification of R & D under consistent conjectures, unpublished (University of Massachusetts at Amherst).