Labour substitution and complementarity among age–sex groups

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We measure substitution in production for major age–sex groups in ten industries. These estimates are important for productivity studies, for modelling derived demand for labour and for formulating policies that deal with anticipated trends in the age–sex composition of the labour force. We use Sato's two-level CES production function to estimate Hicks partial factor price elasticities, with quarterly time-series taken from the Social Security Continuous Work History Sample (1958 to 1975). Own elasticities are generally small and negative but vary considerably across industries. Cross elasticities show complementarity among most groups, except for younger and older females, who are substitutes.

I. INTRODUCTION

Cultural and demographic trends cause the age–sex composition of the labour force to shift over time. In the absence of structural change in production, the substitutability (or complementarity) among age–sex cohorts determines how the relative earnings among age–sex cohorts respond to their relative size. Differential industrial substitutability will determine future likely age–sex employment and productivity patterns. Knowledge of this substitutability is important for productivity studies, for modelling the derived demand for labour in national and regional models and for formulating public policies to deal with anticipated changes in the age–sex composition of the labour force. Substitution among demographic cohorts may itself influence future long-term fertility patterns. A measure of the time required for employment responses to changes in the relative wage rate of a cohort will be instrumental in the analysis of the dynamic adjustment process. Despite the importance of age–sex labour substitution estimates by industry, none has ever been made. A brief survey of the literature follows. The reader is also referred to Hamermesh and Grant (1979) for a very useful critical synthesis of the econometric estimates which exist of substitution among labour aggregates.

Welch (1979) relates changes in relative age-cohort size to the relative aggregate earnings of that cohort. Using Current Population Survey (CPS) data from 1968–75, he finds that increased cohort size depressed earnings for the white male cohorts studied. Freeman (1979) obtains similar results for males, but not for females. The Easterlin cohort-effect fertility model (1968)
uses an assumed negative relationship between cohort size and earnings as the key element. Easterlin (1978) presents an illustrative measure of the depressing effect of the growth of the relative size of the young male cohort on their relative earnings. Similar arguments appear in Easterlin (1968, pp. 114–8; 1973) and Wachter (1976). Wachter (1977) uses the percentage of persons aged 16–34 relative to the total adult population as an explanatory variable in his relative income model for explaining participation on the basis of inferred relative cohort earnings power. His results suggest that relative cohort earnings are inversely related to cohort size. He also reports that 'while the average 20–24-year-old earned almost three-fourths of what a 45–54-year-old worker earned in 1955, he earned only slightly more than half as much in 1975' (p. 558). Grant (1979) estimates a translog cross-section relationship among age groups over standard metropolitan statistical areas (SMSAs) for 1970 and finds very high Allen elasticities of substitution between youths and older workers.

Substitutability by sex has been studied less widely. The first econometric study appears to have been by Weiss (1977). In a study devoted primarily to capital–skill complementarity, he classifies workers by sex, as well as occupation, 'because discrimination precludes employers from treating women as perfect substitutes for men with the same occupational title' (p. 766). The focus of his study, however, is not substitution by sex and therefore he does not report these elasticities.

Freeman (1979) estimates time-series relationships among age and/or sex groups, using a simple CES function and a translog function. The CES estimates necessarily exhibit Hicks complementarity, but his translog estimates show Hicks substitutability between males over 35, and both males under 35 as well as females (pooled by age). Hicks complementarity holds between young males and females and also between capital and the three labour groups. Significance tests, however, are not reported for these estimates.

Grant and Hamermesh (1981) use cross-section data from SMSAs in 1969 to focus on the extent to which women and youths are substitutes in manufacturing. Labour is disaggregated into youths 14–24, adult blacks, adult white females and adult white males. Separability from capital is rejected using translog estimation. Hicks substitutability is found between youths and adult white females, and also between adult white males and adult blacks. Hicks complementarity is found for all other pairs of labour groups, as well as between labour groups and capital. Among labour groups, however, none of the estimated elasticities is significantly different from zero.

Summarizing then, the key estimates to date of substitution among age–sex groups: Grant's (1979) cross-section study finds high Allen substitutability between youths and older cohorts (pooled by sex); the cross-section study by Grant and Hamermesh (1981) finds Hicks substitutability between youths and adult white women; and Freeman's (1979) time-series study finds Hicks substitutability between males over 35 and both males under 35 and females (pooled by age).

Estimates have also been made of substitutability by occupation, education and skill. Bowles (1970), Pascherospolos and Hinchcliffe (1972) and Dougherty (1972) focus on educational classifications, while Griliches (1969), Berndt and Christensen (1974), Fallon and Layard (1975), and Weiss (1977) address the issue of capital–skill complementarity.

The results of such studies could be relevant to estimating age–sex cohort substitutability to the extent that different age–sex cohorts can be characterized by fixed educational and
occupational differences. However, it has been suggested by Edwards (1975) that age and sex may be important per se if sociologically determined authority relationships are used in the organization of the firm. Berndt and Christensen (1974) preface their study of capital–skill complementarity by noting that 'It would be desirable to use data on hours worked by employees classified by personal characteristics such as age, sex, race, education, experience, and health; but such data are extremely difficult to construct' (p. 391).

It is also worth noting that almost all of the research performed pools data over all manufacturing or over the entire economy. The only exception is Weiss (1977), who finds substantial differences across industries in capital–skill complementarity. As already noted, however, he does not report substitutability among labour groups.

In this article, estimates of age–sex substitutability by industry are reported. These estimates are in the form of Hicks partial factor price elasticities for reasons explained in the next section. In the third section, the methods used to develop the required time-series from 1958 to 1975 using the Social Security Continuous Work History Sample (CWHS) are explained. In the subsequent section, the theoretical and empirical reasons for using Sato's two-level CES production function are given. In Sections V and VI, the estimation procedure and results are discussed. Comparisons with the relevant literature are found in the conclusion.

II. HICKS PARTIAL FACTOR PRICE ELASTICITIES

Our analysis is addressed to the effects on the relative wage of an age–sex cohort of an increase in its own supply or in the supply of another age–sex cohort. The limitations on our data put certain restrictions on our analysis. Since we are unable to construct data on nonlabour inputs comparable to our data on labour inputs, we must rest our procedure on the first separability assumption, that of labour inputs from nonlabour inputs. That is, we assume the existence of a labour production function:

\[
L = f(E_1, E_2 \ldots E_n)
\]

where \(E_i\), the employments of the various age–sex cohorts, are considered to be inputs to a linearly homogenous production function \(f\), and where the output, \(L\), enters into commodity production along with nonlabour inputs.\(^2\)

Given this assumption, we can measure the supply effects on the wage of an age–sex cohort,

\(^1\)The reference for this section is Sato and Koizumi (1973).

\(^2\)Berndt and Christensen (1974) and Denny and Fuss (1977) present evidence against such separability, but their results are not directly applicable here. They test the existence of a consistent labour index, comprising skilled and unskilled labour over all manufacturing. Weiss (1977) has shown that their conclusions cannot be sustained when the data are disaggregated by occupation and industry. Grant and Hamermesh (1981) reject separability from capital in a translog cross-sectional study of substitution among youths, adult blacks and adult white males and females. Their data are also pooled over all manufacturing. For a disaggregated study such as ours, it is impossible to get capital data of comparable quality. As Hamermesh and Grant (1979, pp. 523–4) discuss, citing Clark and Freeman (1977) and Berndt (1976), noisy measurements of the price of capital lead to unreliable estimates of substitutability.
relative to labour costs in general, by the Hicks partial factor price elasticities. These elasticities are dual to the Allen partial elasticities of demand, which measure the effects on the relative demand for an age-sex cohort of an increase in its wage or that of another age-sex cohort.

As is well known, the Allen elasticities of substitution and demand, \( \sigma_{ij} \) and \( \eta_{ij} \), can be derived from the unit cost function. In our case, the dual to Equation 1 is

\[
C = g(W_1, \ldots, W_n)
\]

where \( C \), the unit cost of labour output, \( L \), is a function of the wages, \( W_j \), of the various age-sex cohorts. The Allen elasticities of substitution and demand, \( \sigma_{ij} \) and \( \eta_{ij} \), are given by

\[
\sigma_{ij} = \frac{\eta_{ij}}{\theta_j} = \frac{\partial g}{\partial g_j} / \frac{\partial g}{\partial g_j}, \quad \sum_j \eta_{ij} = 0,
\]

where \( \theta_j \) is the distributive share of factor \( j \) (here, the share of labour costs accounted for by group \( j \)), and the subscripts on \( g \) denote partial derivatives of the cost function in Equation 2. These measures describe an experiment where all factor demands and cost (here, all \( E_i \), and \( C \)) vary in response to a single factor price (\( W_j \)), holding constant all other factor prices. To complete the specification of the experiment, we may alternatively conceive of holding output (\( L \)) constant, while varying the levels of factor demands, or, more generally, varying factor intensities relative to output. The Allen elasticities are thus the proper focus, for example, of recent studies on energy-capital complementarity, which consider the impact on capital and labour intensities of exogenous energy price increases.

Our interest, however, is the impact of exogenous factor supply shifts, so the relevant measures are the Hicks elasticities of complementarity and factor prices, \( \phi_{ij} \) and \( \epsilon_{ij} \), given by

\[
\phi_{ij} = \frac{\epsilon_{ij}}{\theta_j} = \frac{\partial f}{\partial f_j} / \frac{\partial f}{\partial f_j}, \quad \sum_j \epsilon_{ij} = 0,
\]

where the subscripts on \( f \) denote partial derivatives of the production function in Equation 1. As shown by Sato and Koizumi (1973), these measures describe an experiment where all factor prices and output (all \( W_j \) and \( L \)) vary in response to a single factor supply (\( E_j \)), holding constant all other factor supplies. We may either conceive of holding unit cost constant while varying the levels of all factor prices, or, more generally, of varying all factor prices relative to unit cost. In particular, we will focus on the factor price elasticities \( \epsilon_{ij} \), which measure the percentage change required in \( W_i \) relative to unit labour cost, to absorb a 1% increase in \( E_j \).

\(^3\)To measure the effects on a cohort's wage relative to the cost of living rather than to labour costs, we would have to take account of the substitution of nonlabour inputs for labour inputs. Thus, an increase in a cohort's supply would reduce its wage relative to the cost of living by more than its reduction relative to the cost of labour. To measure the former effect would, of course, require the data on nonlabour inputs which we lack. Hence, we confine ourselves to the effects on relative wages.

\(^4\)Allen elasticities, \( \sigma_{ij} \) and \( \eta_{ij} \), are also available from the authors. They are related as follows.

Let \( \Lambda = \begin{bmatrix} 0 & 1 & \ldots & 1 \\ \vdots \end{bmatrix} \)

where \( \phi \) is the matrix of Hicks elasticities of complementarity. Then the element in row \( i + 1 \) and column \( j + 1 \) of \( \Lambda^{-1} \) is \( \sigma_{ij}, \theta_j, \theta_i \). (See Sato and Koizumi, 1973, p. 51).
The Hicks measures provide a characterization of complementarity and substitutability which is supplementary to that provided by the Allen measures. As Sato and Koizumi put it, inputs $i$ and $j$ may be classified as Hicks complements or substitutes, with respect to changes in quantities, as $\phi_{ij}, \epsilon_{ij} \geq 0$, while they may be classified as Allen complements or substitutes, with respect to changes in prices, as $\sigma_{ij}, \eta_{ij} \leq 0$. The more unusual cases are Allen complementarity and Hicks substitutability: for $n$ inputs, and, therefore, $n(n-1)/2$ cross elasticities of each variety, there can be no more than $(n-2)(n-1)/2$ instances of Allen complementarity or Hicks substitutability.

III. QUARTERLY EMPLOYMENT AND EARNINGS SERIES

The Social Security Continuous Work History Sample (Bureau of Economic Analysis, US Department of Commerce, 1976) was processed to produce quarterly time-series of employment and earnings for fourteen age-sex groups in each of twenty-eight industries from the second quarter 1957 through the third quarter 1975. These series were subsequently aggregated to the four age-sex groups and twelve industries used in the regression analysis. To limit computation expenses, only every fifth worker on the 1% sample was processed, which resulted in examining some 4513704 annual job records from 310144 employees. They constituted a representative 0.2% sample of all Social Security covered employment in the nation.5

To generate quarterly data series, two limitations of the CWHS file had to be overcome. The first problem was the absence of subsequent earnings in the same year for workers who reached the prevailing Social Security taxable limit. This was corrected by filling in the individual’s largest quarterly earnings that year at and beyond the limit quarter. The correction applied to 16.3% of all workers at its peak in 1965.

The second problem was the absence of information about weeks worked per quarter. This was corrected by assigning permanent or temporary status to each worker in every quarter with nonzero earnings, then accumulating earnings for these two categories separately in each age-sex-industry cell. Temporary status in the current quarter was assigned if either the preceding or following quarter had no earnings or if the current earnings totalled less than 100 hours at the prevailing legal minimum wage. The mean earnings of permanent workers were taken as representative of the wage rate within each cell. Total employment was then measured in permanent equivalents, with temporary employment weighted according to accumulated earnings at the representative wage rate. It should be noted that these imputations were made at the detail of 392 age-sex-industry categories which were subsequently aggregated to the 48 categories used in the regression analysis.

Thus, the employment series count employees, not hours worked. Of course, in the short term, costs of hiring and firing lead firms to respond to relative wages by adjusting hours worked, until the persistence of new relative wage patterns are confirmed. In the long term, however, it is likely that the firm responds by adjusting the number of employees in each age-sex cohort rather than by making a differential adjustment to the workweek for each group. Thus, it is probable that our employment series will show a more sluggish response than might otherwise be measured with data on hours worked.

5All regressions were also run on a 0.1% sample for comparison. This smaller sample yielded qualitatively similar but slightly less significant results.
IV. PRODUCTION FUNCTION

Choice of the Sato function

In choosing a production function, we first considered two options: the CES function, and the translog function. Hamermesh and Grant (1979, p. 521) provide a useful discussion of the choice of functional form. As they point out, the CES function is easiest to estimate, but it constrains all Hicks elasticities of complementarity (and Allen elasticities of substitution) to be equal and also to be positive. On the other hand, the translog function is flexible, but it is difficult to estimate dynamic versions of it, since lagged variables cannot simply be added to the equations. For cross-section studies, such as Grant and Hamermesh (1981) and Grant (1979), translog would appear to be preferable. For time-series analysis, however, such as Freeman (1979), translog may be problematic. Indeed, two of Freeman’s three equations had very low Durbin–Watson statistics, presumably due to finite adjustment speeds which could not be modelled in the translog format.

The function we settled on is a Sato (1967) two-level CES function. This compromise solution has more flexibility than the CES, but less than the translog. As with the CES, finite adjustment speeds can be modelled using lagged dependent variables.

In particular, our four age–sex cohorts assume a $2 \times 2$ structure in the production of aggregate labour:

$$L = \gamma \{ \delta_A (\delta_1 E_1^{-\rho_A} + \delta_2 E_2^{-\rho_A}) \}^{\alpha\beta / \rho_A} + \delta_B (\delta_3 E_3^{-\rho_B} + \delta_4 E_4^{-\rho_B}) \}^{\alpha\beta / \rho_B} \times (-1)^{\beta}, \rho_A, \rho_B, \beta > -1. \quad (5)$$

This function may be instructively re-expressed as

$$L = \gamma \{ \delta_A E_A^{-\beta} + \delta_B E_B^{-\beta} \}^{-1/\beta}, \quad (6)$$

a CES macro function in $E_A$ and $E_B$, where

$$E_A = (\delta_1 E_1^{-\rho_A} + \delta_2 E_2^{-\rho_A})^{-1/\rho_A}; \quad (7)$$

and similarly for $E_B$. That is, $E_A$ and $E_B$ are CES micro functions.

Grouping, separability, and patterns of complementarity

The Sato function in Equation 5 imposes our second separability assumption: groups A and B containing, respectively, labour of cohorts 1 and 2, and 3 and 4, are separable. Apparently some such assumption is the price we have to pay for tractability.

We do retain some flexibility with regard to how we group the four cohorts. Indeed, such flexibility has been the subject of some critical comment. In one of the few applications of the Sato function, Mundlak and Razzin (1971) acknowledge the fact that ‘there is no good empirical procedure to group’ the 11 subaggregates they treat (p. 498). Weiss (1977) eschews the Sato function to avoid ‘arbitrarily’ grouping his inputs, which number between 11 and 15 (p. 767).

Our situation is less problematic since we have confined ourselves to four inputs: males under 35 ($M < 35$), males 35 or over ($M \geq 35$), females under 35 ($F < 35$), and females 35 or over ($F \geq 35$). Of the three possible $2 \times 2$ structures, we find it useful to investigate two: grouping by age and by sex.
Labour substitution and complementarity

The choice of groups has implications for the possible patterns of complementarity and substitutability. To see this, note first that the Hicks elasticities of complementarity can be shown to be

$$\phi_{ij} = (\bar{\rho} + 1)$$

where \(i\) and \(j\) are in different groups. The elasticity between members of the same group is given by

$$\phi_{12} = \{(\theta_\Lambda - 1) (\bar{\rho} + 1) + (\rho_\Lambda + 1)\}/\theta_\Lambda$$

and similarly for \(\phi_{34}\). Equation 8 shows that members of different groups can only exhibit Hicks complementarity \((\phi_{ij}, \varepsilon_{ij} > 0)\). Equation 9 shows that members of the same group can exhibit either Hicks complementarity or the more unusual case, Hicks substitutability \((\phi_{ij}, \varepsilon_{ij} < 0)\).

Since we investigate grouping by age and by sex, we consider the four demographic pairs \((M < 35, M \geq 35)\), \((F < 35, F \geq 35)\), \((M < 35, F < 35)\), and \((M \geq 35, F < 35)\). Each pair is given the chance to show Hicks substitutability in one of the two groupings. If it does so, the case for accepting Hicks substitutability is conditional on that grouping. If it does not, the case for rejecting Hicks substitutability is somewhat stronger.

Cost minimization and lag structure

The estimation of Equation 5 proceeds on the assumption of cost minimization. Sequential cost minimization is legitimate for separable structures such as ours. The optimal factor proportions at the micro level are given by

$$\hat{E}_{i}^* = (E_1/E_2)^* = (\hat{\delta}_\Lambda)^{\sigma_\Lambda} (\hat{\nu}_\Lambda)^{-\sigma_\Lambda}$$

$$\hat{E}_{s}^* = (E_3/E_4)^* = (\hat{\delta}_B)^{\sigma_B} (\hat{\nu}_B)^{-\sigma_B}$$

where \(\hat{\delta}_\Lambda \equiv \delta_1/\delta_2\), \(\hat{\delta}_B \equiv \delta_3/\delta_4\), \(\hat{\nu}_\Lambda \equiv \hat{W}_1/\hat{W}_2\), \(\hat{\nu}_B \equiv \hat{W}_3/\hat{W}_4\), \(\sigma_\Lambda \equiv 1 + \rho_\Lambda\) and \(\sigma_B \equiv 1 + \rho_B\). The optimal factor proportion at the macro level is

$$\hat{E}^* = (E_\Lambda/E_B)^* = (\hat{\delta})^{\sigma} (\hat{\nu})^{-\sigma}$$

where \(\hat{\delta} \equiv \delta_\Lambda/\delta_B\), \(\hat{\nu} \equiv \hat{W}_\Lambda/\hat{W}_B\), \(W_\Lambda \equiv (W_1 E_1 + W_2 E_2)/E_\Lambda\), \(W_B \equiv (W_3 E_3 + W_4 E_4)/E_B\), and \(\sigma \equiv 1 + \rho\).

The Allen elasticities are analogous. Members of different groups can only exhibit Allen substitutability \((\sigma_{ij}, \eta_{ij} > 0)\), while members of the same group can exhibit either Allen substitutability or Allen complementarity \((\sigma_{ij}, \eta_{ij} < 0)\). For this function, Hicks substitutability implies Allen substitutability (but not the converse) and Allen complementarity implies Hicks complementarity (but not the converse).

We have taken the macro decision to be governed by short-run unit costs of producing \(E_\Lambda\) and \(E_B\), which are based on the actual micro factor proportions \(E_1/E_2\) and \(E_3/E_4\). Alternatively, one might argue that the macro manager is guided by long-run unit costs, based on the fully adjusted micro proportions \((E_1/E_2)^*\) and \((E_3/E_4)^*\).
We assume each of the three proportions adjusts with geometric lag, at a common adjustment speed \( \lambda \). Thus

\[
\begin{align*}
\dot{E}_{A,t} &= (\delta_A)_{\lambda A}(\dot{W}_{A,t})^{-\lambda A}(\dot{E}_{A,t-1})^{1-\lambda} \\
\dot{E}_{B,t} &= (\delta_B)_{\lambda B}(\dot{W}_{B,t})^{-\lambda B}(\dot{E}_{B,t-1})^{1-\lambda} \\
\dot{E}_t &= (\delta)_{\lambda}(\dot{W}_t)^{-\lambda}(\dot{E}_{t-1})^{1-\lambda}
\end{align*}
\]  

(12)

Finally, to make our left-hand side directly observable (\( \dot{E} \) is not), multiply through Equation 12 by \( \dot{W}_{A,t} \), \( \dot{W}_{B,t} \), and \( \dot{W}_t \) to yield

\[
\begin{align*}
\dot{y}_{A,t} &= W_1 E_1 / W_2 E_2 \\
\dot{y}_{B,t} &= W_3 E_3 / W_4 E_4 \\
\dot{y}_t &= (W_1 E_1 + W_2 E_2) / (W_3 E_3 + W_4 E_4) = (\delta)_{\lambda}(\dot{W}_t)^{-\lambda}(\dot{E}_{t-1})^{1-\lambda}
\end{align*}
\]  

(13)

V. ESTIMATION

The two major problems encountered in econometric studies of markets are the interrelated questions of identification and simultaneous equation bias. Here the first problem is solved by noticing that supply of an age–sex cohort to an industry will shift when the average wage of the age–sex cohort shifts in other industries, while demand from an industry for the particular age–sex cohort in question will shift when the average wage of other age–sex cohorts in the industry shift. Thus industry wage and cohort wage can be used to identify the demand and supply curves, respectively. The problem of biased estimates due to simultaneity is solved by using instrumental variables based on a first stage where only information exclusive of the cohort and the industry in question is used.

More specifically, a two-stage least squares technique is used for each industry. In the first stage, the instrumental variables for the \( I \)th industry are found as follows

\[
\begin{align*}
\bar{W}_{IC} &= \pi_0 + \pi_1 W_I + \pi_2 E_I + \pi_3 W_C + \pi_4 E_C + \text{error term}
\end{align*}
\]  

(14)

where \( W_I = \text{Wage in industry} \ I \) excluding \( W_{IC} \); \( E_I = \text{Employment in industry} \ I \) excluding \( E_{IC} \); \( W_C = \text{Wages of cohort} \ C \) excluding \( W_{IC} \); and \( E_C = \text{Employment of cohort} \ C \) excluding \( E_{IC} \).

Then in the second stage \( \bar{W}_{IC} \) is used as an instrument in the explanatory variable set in Equation 13. The fact that the regressions are performed by industry using the specification in Equation 14 above identifies the system.

In the second stage, the three equations which comprise Equation 13 must be jointly estimated, since they have \( \lambda \) in common. We assume that independently distributed error terms enter additively after taking logs. The system is nonlinear, since \( \ln \dot{W}_t \) and \( \ln \dot{E}_{t-1} \) are constructed from nonlinear combinations of the data and the unknown parameters. We use an iterative least squares method to find our point estimates of the parameters of the production function. The Hicks factor price elasticities are linear combinations of the parameters weighted by various shares (\( \theta \)'s). We evaluate those shares at the sample means. The asymptotic covariance matrix

\(^9\text{See Maddala (1977), also Berndt et al. (1974).}\)
for the parameters of the production function is used to derive standard errors for the Hicks factor price elasticities, conditional on the shares at the sample means.

VI. RESULTS AND INTERPRETATION

Tables 1 and 2 show our results for the Hicks factor price elasticities. The function underlying Table 1 groups by sex, while that underlying Table 2 groups by age.

The $R^2$ we report is based on the goodness of fit for the employment ratios of $M < 35$, $M \geq 35$, and $F < 35$ relative to our 'numeraire', $F \geq 35$. The $R^2$ were uniformly high, so they provide us with no evidence in favour of one grouping as opposed to another. We therefore consider both tables.

Of the twelve industries, only hospitals and personal services were problematic. When grouping by age, personal services failed to converge, and hospitals did not show concavity. When grouping by sex, these two industries failed to provide any elasticities significantly different from zero. For hospitals, it may be appropriate to doubt the assumption of cost minimization. For these reasons, we leave these two industries out of the ensuing discussion.

The remaining ten industries yielded some interesting results. The Hicks elasticities consider the impact of increased employment by some cohort on its own wage (relative to unit labour cost) and on the wage of other cohorts. Thus, for example, according to Table 1, a 1% increase in younger male employment in agriculture would induce a fall in their wage of 0.10%, while it would raise the wage of older males by 0.05% and of all females by 0.04%. We will first discuss our estimates of own elasticities, then the cross elasticities.

Overall, our estimates of the own elasticities are rather small. Of the 80 estimates, only one exceeds unity in magnitude, and the median is about $-0.20$. These elasticities suggest that a growing age-sex cohort does experience some difficulty in the labour market, but not as great as that implied by some demographic models.

One might expect that some cohorts are harder to absorb than others. On this point, the evidence is mixed. When grouping by sex, own elasticities do not appear to differ systematically by cohort. When grouping by age, however, the median own elasticities are $-0.38$ for older females, $-0.26$ for younger females, $-0.19$ for older males, and $-0.18$ for younger males. If the age grouping is correct, this would suggest more difficult absorption for females, particularly older females.

We may also consider ease of absorption by industry. Both sets of estimates indicate that the regulated industries have high own elasticities for all cohorts, which may suggest strict categorization of jobs by age and sex. This result would be consistent with the neoclassical hypothesis that regulated industries, protected from competitive pressures, have been particularly prone to institutional segmentation. Finance is another regulated industry with relatively high own elasticities.

Both of our sets of estimates confirm that construction has been relatively inhospitable to increased participation by females. If the estimates based on grouping by sex are correct, agriculture is another problem area for females. If our estimates based on grouping by age are correct, then, as noted above, older females have generally high own elasticities, with the highest occurring in business services.
<table>
<thead>
<tr>
<th>Industry</th>
<th>Males &lt; 35</th>
<th>Males ≥ 35</th>
<th>Females &lt; 35</th>
<th>Females ≥ 35</th>
<th>λ</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Own</td>
<td>M ≥ 35</td>
<td>F</td>
<td>Own</td>
<td>F ≥ 35</td>
<td>M</td>
</tr>
<tr>
<td>1. Agriculture</td>
<td>-0.10ᵇ</td>
<td>0.05ᵇ</td>
<td>0.04ᵇ</td>
<td>-0.07ᵇ</td>
<td>0.08ᵇ</td>
<td>0.07ᵇ</td>
</tr>
<tr>
<td>2. Construction</td>
<td>-0.21ᵇ</td>
<td>0.10ᵇ</td>
<td>0.31ᵃ</td>
<td>-0.14ᵇ</td>
<td>0.17ᵇ</td>
<td>0.52ᵇ</td>
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<tr>
<td>3. Durables</td>
<td>-0.22ᵇ</td>
<td>0.09ᵇ</td>
<td>0.03</td>
<td>-0.12ᵇ</td>
<td>0.19ᵇ</td>
<td>0.06</td>
</tr>
<tr>
<td>4. Nondurables</td>
<td>-0.20ᵇ</td>
<td>0.08ᵇ</td>
<td>0.08ᵇ</td>
<td>-0.12ᵇ</td>
<td>0.16ᵇ</td>
<td>0.16ᵇ</td>
</tr>
<tr>
<td>5. Regulated</td>
<td>-0.46ᵇ</td>
<td>0.32ᵇ</td>
<td>0.14ᵇ</td>
<td>-0.37ᵇ</td>
<td>0.41ᵇ</td>
<td>0.19ᵇ</td>
</tr>
<tr>
<td>6. Wholesale</td>
<td>-0.09ᵇ</td>
<td>0.04ᵇ</td>
<td>0.04</td>
<td>-0.05ᵇ</td>
<td>0.07ᵇ</td>
<td>0.08</td>
</tr>
<tr>
<td>7. Retail</td>
<td>-0.13ᵇ</td>
<td>0.05ᵇ</td>
<td>0.06ᵇ</td>
<td>-0.11ᵇ</td>
<td>0.07ᵇ</td>
<td>0.09ᵇ</td>
</tr>
<tr>
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<td>0.06ᵇ</td>
<td>-0.17ᵇ</td>
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</tr>
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<td>12. Business</td>
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<td>0.02</td>
<td>-0.06ᵃ</td>
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<td></td>
<td></td>
<td>0.03ᵇ</td>
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</table>

ᵃStatistically significant at the 5% level (one-tail test for columns 1, 3, 4, 6, 7, 9, 10 and 12; two-tail test for columns 2, 5, 8 and 11).
ᵇStatistically significant at the 1% level (with one- and two-tail tests as noted above).
Table 2. Hicks factor price elasticities: grouping by age

<table>
<thead>
<tr>
<th>Industry</th>
<th>Males &lt; 35</th>
<th>Females &lt; 35</th>
<th>Males &gt; 35</th>
<th>Females &gt; 35</th>
<th>Own</th>
<th>F &lt; 35</th>
<th>All ≥ 35</th>
<th>Own</th>
<th>M &lt; 35</th>
<th>All ≥ 35</th>
<th>Own</th>
<th>F ≥ 35</th>
<th>All &lt; 35</th>
<th>Own</th>
<th>M &gt; 35</th>
<th>All &lt; 35</th>
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<tr>
<td>4. Nondurables</td>
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<td>0.08⁴</td>
<td>-0.24⁴</td>
<td>0.01⁴</td>
<td>0.01⁴</td>
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<td>0.02⁴</td>
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<td>6. Wholesale</td>
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<td>0.06⁴</td>
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<td>7. Retail</td>
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<td>-0.39⁴</td>
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</table>

*Statistically significant at the 5% level (one-tail test for columns 1, 3, 4, 6, 7, 9, 10 and 12; two-tail test for columns 2, 3, 8 and 11).

bStatistically significant at the 1% level (with one and two tail tests as noted above).

NA: not available due to inability to solve.

NC: estimates failed concavity and negative λ.
The ease of absorption in an industry is affected not only by own elasticities, but also by the industry's speed of adjustment. In general, the adjustment speeds are rather slow: the median of our 20 estimates implied that even if the requisite wage cut was immediate, only 12% of the adjustment would be completed within one quarter and only 40% within a year. By both groupings, restaurants adjust most quickly, completing 27% or 36% of the adjustment within one quarter, and 72% or 83% within a year. This would be consistent with the high turnover rate in that industry.\(^\text{10}\)

Now let us consider the cross elasticity estimates. As discussed in Section IV, the Sato function will not allow cohorts to exhibit Hicks substitutability unless they are grouped together. Table 2 shows that when grouping by age, males and females generally failed to show Hicks substitutability: only 3 of the 20 independent cross elasticities within age groups (\(\varepsilon_{ij}\) has the same sign as \(\varepsilon_{kl}\)) were estimated to be negative, and none significantly so. Older males and females appear to be particularly complementary with each other, relative to younger workers: in 9 out of 10 industries an increase in the supply of older workers of one sex increases the wage of the older workers of the other sex, not only relative to unit labour costs (which includes the wage cut of the enlarged cohort), but also relative to younger workers. For example, in the durables industry, a 1% increase of older male employment raises the wage of older females relative to younger workers by \(0.52 - 0.11 = 0.41\%\), and, conversely, a 1% increase of older female employment raises the wage of older males relative to younger workers by \(0.16 - 0.04 = 0.12\%\).

The estimates based on grouping by sex in Table 1 show a striking contrast. While Hicks complementarity remains the rule between younger and older males (except in the two problematic industries), younger and older females show a pattern of Hicks substitutability. Except for agriculture, all cross elasticities between females are negative, though significant only for finance. An increase in the supply of females of one age cohort reduces the wage of females of the other age cohort, relative to unit labour costs, and, therefore, particularly relative to males. For example, in the construction industry, a 1% increase of younger female employment reduces the wage of older females by 0.24% relative to unit labour costs and by 0.26% relative to males. Conversely, a 1% increase of older female employment reduces the wage of younger females by 0.41% relative to unit labour cost and by 0.44% relative to males.

The patterns specific to Tables 1 and 2, of course, are conditional on the respective grouping assumptions. Some patterns, however, are robust with respect to the grouping assumption. Thus, as noted above, Hicks complementarity is the dominant relationship between younger males and younger females, between younger males and older males, and between older males and older females, all regardless of grouping. (For completeness, we may note that by omitting to group younger males with older females and younger females with older males, we have forced complementarity on those two pairs.) Furthermore, although Hicks substitutability between younger and older females can only be posited conditional on grouping by sex, we can establish the following unconditional pattern: an increase in older female employment reduces the wage of younger females relative to older males in 9 out of 10 industries, regardless of

\(^{10}\)See Bureau of Labor Statistics, Special Labor Force Report No. 235, Tables F and G. Whereas a quarter of all males and a third of all females started their current jobs within one year prior to January, 1980, within food services the corresponding ratios are two-thirds for males and a half for females.
grouping (though the exceptional industry differs by grouping). Thus older females might be classified unambiguously as relatively Hicks substitutable with younger females, in reference to older males. (The converse does not hold when grouping by age.)

What are we to make of these provocative patterns? The estimated Hicks complementarity between males and females of both age cohorts is consistent with the literature on sexual segmentation of the labour force. The estimated complementarity between younger and older males is consistent with segmentation by age, due either to institutional conditions such as seniority, or to the effect of experience on the accumulation of human capital. The fact that complementarity cannot be established between younger and older females suggests that age segmentation may be confined to males. This would lead us to distinguish our interpretation of younger male–female complementarity from that of older male–female complementarity: the poor substitutability between older males and females may be due to different skills, as the human capital school might suggest; but the poor substitutability between younger males and females, before either group has acquired much experience, may reflect an institutional decision with respect to which sex will be allowed to acquire which types of skills.

VII. Summary

This study has produced estimates of Hicks factor price elasticities for four age–sex cohorts in ten industries. These estimates have the following salient characteristics:

1. Own elasticities are negative, but relatively small, on the order of $-0.20$. This is consistent with the cross-section translog estimates of Grant and Hamermesh (1981) who found comparable elasticities for youths, adult males and adult white females. Freeman (1979) also finds low own elasticities for his CES function, but not for his translog. As noted earlier, however, his time-series analysis for translog assumes infinite adjustment speed, which appears to be untenable, given his Durbin–Watson statistics and our $\lambda$s.

2. If the age grouping is correct, absorption in the labour force may be somewhat more difficult for females, particularly older females. This is consistent with Grant and Hamermesh (1981), who estimate the factor price elasticity of adult white females as $-0.19$ vs $-0.03$ for youths. Also, although Freeman’s translog factor price elasticities appear large, he, too, finds that of women ($-0.71$) substantially larger than younger and older men ($-0.38$ and $-0.49$).

3. Industries differ with respect to absorptive capacity: some require relatively large wage cuts for any age–sex cohort (regulated industries), some only require large wage cuts for women (construction). Most have low adjustment speeds, except for restaurants.

4. Our cross elasticities imply that Hicks complementarity is the typical relationship between males and females of both age cohorts, and between younger and older males. This is consistent with Hamermesh and Grant (1979), who find complementarity between adult white males and females, as well as between youths and adult white males. Our results are less consistent with Freeman (1979), who finds substitutability between younger and older males.

5. We find notable evidence of Hicks substitutability between younger and older females. Neither Freeman (1979) nor Grant and Hamermesh (1981) examine this elasticity, since Freeman pools all women, and Grant and Hamermesh pool all youths. Grant and Hamermesh do find Hicks substitutability between youths and adult white females.
If the structure of US industry remains the same with regard to the treatment of workers in age–sex categories, the results shown can be used in labour market modelling. To the extent that legal, social, attitudinal, educational and other characteristics that divide sex roles are reduced, substitution by sex should become easier. The same can be said for age segmentation, except to the extent that there is a necessary association between age and length of experience.

ACKNOWLEDGEMENTS

David Ehrlich prepared the basic data files. C. Lon Chen and Sinan Koont served as research associates and made a substantial contribution to executing the estimation algorithms. Funding support was provided by the Massachusetts Division of Employment Security and other state agencies through the Massachusetts Economic Policy Analysis (MEPA) Project.

REFERENCES

Labour substitution and complementarity


