Keynesian Models of the Short Run and the Steady State

By

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1. Introduction and Summary

The distinguishing feature of a Keynesian model is an investment function which is independent of the savings function, i.e. the abandonment of Say's Law. Short-run models have accommodated this additional equation by omitting one or the other of the two neoclassical labor market conditions: labor-supply pricing (Keynes, 1936) or marginal productivity wage payments (Patinkin, 1965). Long-run models, however, have found it difficult to reconcile a Keynesian investment function with a steady-state configuration for the employment rate, as indicated by Robinson's (1962) various "ages". In the present paper, we propose such a reconciliation, in models which rely heavily on a point by Matthews (1954), that the average propensity to save depends positively on the employment rate, for a variety of reasons, discussed below.

We begin in Section 2 by laying out the elements of such a model: a neoclassical production function; a savings function which exhibits the Matthews effect, as well as the classical/Cantabrigan distribution effect; an investment function governed by "productive capacity and profitability", the two elements stressed by Malinvaud (1980, 1982, 1983, 1984); marginal productivity wage payments and/or labor-supply pricing. Section 3 reviews the short-run and steady-state behavior of the neoclassical model.

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developing some interesting implications of incorporating the Matthews effect. Sections 4 and 5 present the short-run and steady-state behavior of our Keynesian model, under the respective assumptions of marginal productivity wage payments and labor-supply pricing. For all of these models, we analyze the short-run and steady-state responses of employment, wages, and capital-intensity with respect to changes in “animal spirits”, “thrift”, the exogenous component of the labor supply price, and the population growth rate. These results, which form the bulk of the paper, are summarized in Figs. 1–4 and in the concluding remarks.

2. Elements of the Models

In this section, we lay out the elements of our models: production function, savings and investment functions, and labor market conditions. Notation, for this and subsequent sections, is given in Table 1.

<table>
<thead>
<tr>
<th>L</th>
<th>employment</th>
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<tbody>
<tr>
<td>P</td>
<td>population</td>
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<td>K</td>
<td>capital stock</td>
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<td>Q</td>
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<td>S</td>
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<td>e</td>
<td>L/P, employment rate</td>
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<td>s</td>
<td>S/Q, average propensity to save</td>
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<td>v</td>
<td>K/Q, capital/output ratio</td>
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<tr>
<td>k</td>
<td>K/L, capital/labor ratio</td>
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<td>c</td>
<td>K/P, capital/population ratio</td>
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<td>m</td>
<td>Q/L, marginal productivity of labor</td>
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<td>w</td>
<td>real wage rate</td>
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<tr>
<td>h</td>
<td>S/K, growth rate of the supply of capital</td>
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<td>g</td>
<td>I/K, growth rate of the demand for capital</td>
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<td>n</td>
<td>growth rate of the population</td>
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<td>T</td>
<td>shift parameter for thrift</td>
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<td>A</td>
<td>shift parameter for animal spirits</td>
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<td>w</td>
<td>shift parameter for labor supply price</td>
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(a) Production Function

Production is characterized by a well-behaved production function,

\[ Q = F(K, L). \]  (2.1)
subject to constant returns, and enjoying no technical progress. We will often find it convenient to express the marginal productivity of labor and the capital/output ratio as functions of the capital/labor ratio, \( k \):

\[
m = \frac{\partial Q}{\partial L} = m(k), \quad m'(k) > 0, \\
v = \frac{K}{Q} = v(k), \quad v'(k) = \frac{mv^2}{k^2} > 0.
\]

(b) Savings Function

The savings behavior of the economy is formulated in terms of savings per unit of capital,

\[
h \equiv S/K = h(k, w, e; T),
\]

where subscripts denote partial derivatives. To motivate this formulation, recall Harrod's (1939) observation that

\[
s/v.
\]

where \( s / v \) is the average propensity to save. Let

\[
s = s(w, e; T), \quad \partial s/\partial w < 0, \quad \partial s/\partial e > 0, \quad \partial s/\partial T > 0.
\]

The partial with respect to the wage represents the familiar classical/Cantabrigian distribution effect (e.g., Kaldor, 1960), whereby a rise in the share of labor, at the expense of capital, reduces the propensity to save. If the share of labor varies with the wage rate, it then follows that \( \partial s/\partial w < 0 \).

Second, we allow the employment rate to have a positive influence on the propensity to save, as Matthews (1954) has suggested, for a variety of reasons. In the short run, we may expect the unemployed to dissave by drawing down assets. More importantly for our purposes, there is likely to be a long-run effect, as the unemployed, with a high propensity to consume, are supported by transfers (voluntary, familial, or public) from employed labor and capital.\(^1\) It should be noted that this formulation is general

\(^1\) On the other hand, a high unemployment rate might lead to precautionary saving among the employed. This effect, however, is likely to be short-run in nature, and we assume it is dominated, so the positive Matthews effect prevails.
enough to cover both involuntary unemployment, as Matthews had in mind, and voluntary unemployment, as in neoclassical models (i.e., low labor force participation rate is postulated to depress savings).

The shift parameter $T$ will allow us to analyze the effects of exogenous changes in "thrift".

Finally, since $i''(k)>0$, by (2.3), the Harrod equation (2.5) immediately implies $h_k<0$ in (2.4). The economic understanding of this is enhanced by first observing that in the short run, with a given capital stock, $k = K/L$ moves inversely with employment, $L$, and, hence, output. Therefore, $h_k<0$ signifies that, with a given propensity to save, savings increase with output, the familiar Keynesian phenomenon.

(c) Investment Function

The investment equation is also formulated in terms of investment per unit of capital:

$$g = I/K = G(v(k), w; A) = g(k, w; A),$$

$$g_k = G, v'(k) < 0, g_s < 0, g_A > 0.$$  (2.7)

The investment function $G$ proposes to capture the two elements which Malinvaud has been stressing (1980, 1982, 1983, 1984), "productive capacity and profitability". The first two determinants, $v(k)$ and $w$ suffice to indicate whether the existing capital/output ratio exceeds or falls short of the cost-minimizing one. If the existing capital/output ratio is unduly high, this clearly discourages investment, $G_s<0$. It is convenient and legitimate to use the chain rule to express this relationship through the variable $k$, so $g_k$ is negative as well.

The second variable, $w$, raises the cost-minimizing capital/output ratio, which tends to raise investment. However, $w$ also governs the "profitability" of investment. To quote Malinvaud (1984, pp. 49–50),

Clearly, profitability must be understood as a measure of disequilibrium existing in the price system, the same being true of Tobin's $q$. It may be defined, for instance, as the pure profit rate, namely the excess of the real profit rate . . . over the real interest rate . . . The frequent existence of such disequilibria in the price system is an observed fact.

In other work (Costrell, 1983, 1986) one of us has investigated the
implications of the equilibration of the interest rate to the profit rate, eliminating pure profits. For the purposes of the present paper, however, we follow Malinvaud in letting this disequilibrium persist, e.g. by implicitly assuming an unchanging interest rate. A rise in the wage, therefore, reduces the pure profit rate. Hence, a rise in the wage has two conflicting effects on investment: it raises the cost-minimizing capital/output ratio; and it reduces pure profitability. Accordingly, the net effect, \( g_w \), is of ambiguous sign. We shall proceed on the assumption that the Malinvaud profitability effect dominates, \( g_w < 0 \), although conclusions based on this should be treated with some caution.

Finally, we include a shift variable, \( A \), to allow us to analyze the effects of an improvement in "animal spirits".

### (d) Labor Market Equations

The neoclassical labor market satisfies the marginal productivity and labor supply price conditions:

\[
\begin{align*}
w &= m(k), & m'(k) > 0, & (2.8) \\
\bar{w} &= w'(e) + \tilde{w}, & w'(e) > 0, & (2.9)
\end{align*}
\]

where \( w(e) \) represents the rising supply price of labor, and where \( \tilde{w} \) is an exogenous shift factor.

In Keynesian models, one or the other of these conditions is typically dropped, but there is no consensus on the choice. Keynes (1936), in a concession to Marshall, let the wage follow the marginal productivity of labor, which moves countercyclically (inversely to \( L \), given \( K \)). Critics presented evidence that real wages move procyclically (Dunlop, 1938; Tarshis, 1939). Patinkin (1965) concurred in dropping the marginal productivity assumption by arguing that the Keynesian labor demand curve is truncated due to the limit of effective demand. Keynes himself (1939) was quite ready to drop the marginal productivity condition, concluding that in the short run, at least, we may do well to take the wage as constant. More recent empirical work (Bodkin, 1969; Canzoneri, 1978; Neftci, 1978; Otani, 1978; Geary and Kenan, 1982; Sachs, 1983; Schor, 1985) has proven inconclusive.

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2 The labor supply price could, alternatively, be interpreted as the wage which labor can obtain as a result of its bargaining power, where the bargaining power improves with the employment rate. These alternative interpretations are difficult to distinguish observationally.
In our Keynesian models of Sections 4 and 5, we shall take an ecumenical approach, assuming, respectively, marginal productivity wage payments, and labor supply pricing (of which the exogenous wage is a special case). This will allow us to ascertain which results are robust with respect to the labor market formulation, and which results are sensitive. Before doing so, however, we shall briefly review the neoclassical model, allowing it to incorporate the Matthews effect.

3. The Neoclassical Model

In this section we present a familiar neoclassical growth model, primarily for comparison with the Keynesian models to follow. We begin with the analysis of short-run equilibrium and comparative statics, followed by steady-state equilibrium, stability analysis, and comparative dynamics.

(a) Short-Run Equilibrium and Comparative Statics

Neoclassical short-run equilibrium is fully characterized by the labor market conditions, reproduced here:

\[ w = m(k = c/e), \quad (3.1) \]
\[ w = w(e) + \bar{w}, \quad (3.2) \]

where we have used the identity

\[ k = K/L = (K/P)/(L/P) = c/e \quad (3.3) \]

to substitute for \( \bar{w} \). We assume these three equations suffice to solve for \( e, w, \) and \( k \), since the capital/population ratio, \( c \), is predetermined in the short run.

The comparative statics with respect to \( \bar{w} \) and \( c \) are familiar, but are presented for purposes of comparison with the steady state and with Keynesian models:

\[ \frac{de}{d\bar{w}} = \frac{-1}{|J|} < 0, \quad \frac{dw}{d\bar{w}} = \frac{m'k}{e|J|} > 0, \quad \frac{dk}{d\bar{w}} = \frac{k}{e|J|} > 0, \]
\[ \frac{de}{dc} = \frac{m'}{e|J|} > 0, \quad \frac{dw}{dc} = \frac{w'm'}{e|J|} > 0, \quad \frac{dk}{dc} = \frac{w'}{e|J|} > 0, \]

where \( |J| = w' + m'k/e > 0 \).
An exogenous increase in the labor supply price increases the wage and reduces the employment rate, which increases the capital-intensity in the short run. A rise in \( c \), due to a one-shot increase in the capital stock or decrease in the population, increases the wage, employment rate, and capital-intensity.

It should also be noted that "thrift" has no short-run effects in the neoclassical model; "animal spirits" have no role at all in the neoclassical model; and the population growth rate has no short-run effects in any of the models considered in this paper.

(b) **Steady State, Stability, and Comparative Dynamics**

Short-run equilibrium is conditional on the predetermined capital/population ratio, \( c \). Over time, however, this ratio grows at the rate

\[
\dot{c} = \dot{c} = h(k = c/e, w, e; T) - n. \tag{3.4}
\]

where \( \dot{c} \) denotes growth rate, and where \( n \) is the (natural) rate of population growth.

The steady-state condition, of course, is that \( \dot{c} = 0 \), or \( h = n \). If steady state exists, the stability condition is

\[
\frac{d\dot{c}}{dc} = \frac{dh}{dc} = \frac{\dot{c}h}{c} + (\frac{\dot{c}h}{ce} \cdot \frac{de}{dc}) + (\frac{\dot{c}h}{cw} \cdot \frac{dw}{dc}) < 0 \tag{3.5}
\]

at the steady state. In the present model, we have

\[
\frac{d\dot{c}}{dc} = (h_k w' + h_m m' w' + h_e m')/e |J|. \tag{3.6}
\]

The first term in parentheses is stabilizing (negative), since \( h_k < 0 \). This represents the Solow (1956) mechanism: for \( h > n \), \( c \) rises, which increases the capital-intensity \( k \) (recall \( dk/dc > 0 \)), which in turn reduces savings per unit of capital (\( h_k < 0 \)), thus reducing the growth rate of \( c \). The second term is also stabilizing since, as \( c \) rises, so does the wage (\( dw/dc > 0 \)), which reduces savings by the classical/Cantabrigian distribution effect (\( h_w < 0 \)), again reducing the growth rate of \( c \). The last term, however, is destabilizing, since, as \( c \) rises, so does the employment rate (\( de/dc > 0 \)), which
increases savings by the Matthews effect ($h_c > 0$). In the limiting case of perfectly elastic supply of labor, $w'(e) \equiv 0$, the Matthews effect renders the model unstable. Conversely, in the limiting case of perfectly inelastic supply of labor, $w'(e) \to \infty$, typically assumed in growth theory, stability obtains.

Assuming stability, we can analyze the behavior of our steady-state variables in response to exogenous shocks:

\[
\begin{align*}
\frac{de^*}{dT} &= \frac{m'h_t}{|J^*|} > 0, \\
\frac{dw^*}{dT} &= \frac{m'w'h_t}{|J^*|} > 0, \\
\frac{dk^*}{dT} &= \frac{w'h_t}{|J^*|} > 0, \\
\frac{de^*}{dn} &= -\frac{m'}{|J^*|} < 0, \\
\frac{dw^*}{dn} &= -\frac{m'w'}{|J^*|} < 0, \\
\frac{dk^*}{dn} &= -\frac{w'}{|J^*|} < 0, \\
\frac{de^*}{d\bar{w}} &= \frac{h_t + h_c m'}{|J^*|} < 0, \\
\frac{dw^*}{d\bar{w}} &= -\frac{m'h_t}{|J^*|} < 0, \\
\frac{dk^*}{d\bar{w}} &= -\frac{h_t}{|J^*|} < 0,
\end{align*}
\]

where $|J^*| = -e|J| \frac{d\bar{c}}{dc} > 0$, by the stability condition.

Fig. 2 on p. 369 depicts the response of the system to an increase in thrift. While an increase in thrift has no short-run effect in the neoclassical model, it is well-known that it increases the steady-state capital-intensity and wage. Furthermore, it increases the employment rate, by drawing more workers up the labor supply curve.

Similarly, as Fig. 4 on p. 370 shows, an increase in the population growth rate has no short-run effect, but it reduces the steady-state capital-intensity and wage, thereby pushing the employment rate down the labor supply curve.

Finally, consider an upward shift (equivalently, a leftward shift) of the labor supply curve. As we saw before, this reduces employment in the short run, and we can also show that in the steady state, employment falls even further. This reduction in the steady-state employment rate reduces the average propensity to save, by the Matthews effect. Hence, this acts much like a reduction in thrift, reducing the steady-state capital-intensity and wage. That is, pushing up the supply price of labor yields short-run wage gains, which are reversed in the steady state, as indicated in Fig. 3 on p. 370, leaving workers worse off than when they began.

To summarize, the main purpose of this section has been to present a familiar neoclassical growth model, for comparison with the Keynesian models to follow. However, by introducing the Matthews effect, we have found two (related) novel results. First, a
rise in the supply price of labor reduces the steady-state wage. Second, the Matthews effect is destabilizing (though perhaps not decisively). Interestingly enough, we shall see below that in our Keynesian models, the Matthews effect is stabilizing, and critically so.

4. A Keynesian Model: Marginal Productivity Wages

In this and the following section, we construct Keynesian models, integrating short-run and steady-state analysis. The model of the present section follows Keynes in dropping the labor supply condition, and retaining the marginal productivity condition; Section 5 does the reverse.

The short-run Keynesian equilibrium condition equates savings and investment,

\[ h(k \equiv c/e, w; e; T) = g(k \equiv c/e, w; A), \quad (4.1) \]

where the identity (3.3) has been substituted into (2.4) and (2.7). The marginal productivity condition is

\[ w = m(k \equiv c/e), \quad m'(k \equiv c/e) > 0. \quad (4.2) \]

We assume this system solves for \( e, w, \) and \( k, \) conditional on \( A, T, \) and \( c. \)

The comparative static derivatives, with presumptive signs are:

\[
\begin{align*}
\frac{de}{dA} &= \frac{g_A}{|J|} > 0, \\
\frac{dw}{dA} &= \frac{-m'k g_A}{e|J|} < 0, \\
\frac{dk}{dA} &= \frac{-k g_A}{e|J|} < 0, \\
\frac{de}{dT} &= \frac{-h_T}{|J|} < 0, \\
\frac{dw}{dT} &= \frac{m'k h_T}{e|J|} > 0, \\
\frac{dk}{dT} &= \frac{k h_T}{e|J|} > 0, \\
\frac{de}{dc} &= \frac{|J| - h_c}{k|J|} > 0, \\
\frac{dw}{dc} &= \frac{m'h_c}{e|J|} > 0, \\
\frac{dk}{dc} &= \frac{h_c}{e|J|} > 0,
\end{align*}
\]

where \(|J| = h_c + (g_k - h_k) k/e + (g_a - h_a) m'k/e > 0.\)

To sign \(|J|,\) first note that \(de/dA\) is essentially the Keynesian multiplier, while \(de/dT\) represents the paradox of thrift. For these derivatives to have the appropriate signs (positive and negative, respectively), we require that \(|J|\) be positive, as indicated. This avoids the explosive process associated with a negative multiplier.
Consider the three terms in $|J|$ in more detail. The first one has the appropriate sign, since $h_1 > 0$. This represents the fact that an increase in aggregate demand, which raises employment, increases the propensity to save, *ipso facto*, through the Matthews effect, and thereby dampens the multiplier. The second term is equivalent to the simple Keynesian condition that the marginal propensity to save out of income exceeds the marginal propensity to invest, and we shall assume that it too has the appropriate sign, $\frac{g_1 - h_1}{\varphi} > 0$. The last term represents the wage effects on the multiplier: as employment expands, moving us down the marginal productivity curve, the wage falls, which raises the propensities to save (by the classical/Cantabrigian distribution effect) and to invest (by the Malinvaud profitability effect). The effect on the multiplier depends on which propensity rises more, a point on which we remain agnostic. We shall, however, assume that it cannot outweigh the simple Keynesian condition, such that the second and third terms together, $|J| - h_1$, remain positive. That is, we assume that the multiplier would be positive, even without the Matthews effect (although, as we shall see, the Matthews effect is quite critical to the long-run stability of this model).

Before discussing the short-run effects any further, we shall lay out the steady-state effects. If there exists a unique steady-state configuration for $e^*$ and $k^*$, it is given by

$$g(k^*, w = m(k^*); \varphi) = n,$$  \hspace{1cm} (4.3)

$$h(k^*, w = m(k^*), e^*; \varphi) = n.$$  \hspace{1cm} (4.4)

It should immediately be noted that if there were no Matthews effect, $e^*$ would not appear in either of these two equations, so we would have two equations for one unknown, $k^*$. If so, there would in general be no steady state.\(^4\) In this case, the short-run model (4.1, 4.2) would determine equilibrium $k$ such that $g = h$, and this growth rate of capital would persist indefinitely. It would only be a fluke if it coincided with the growth rate of the population, $\varphi$.\(^5\) If $g = h < n$ at the equilibrium $k$, then $c$ and $e$ decline in step, asymptotic.

---

\(^1\) This will help sign derivatives in section 5.

\(^4\) Of course, if the Matthews effect is weak, there may still be no steady state.

\(^5\) In Harrodian terms, it would only be a fluke if the “warranted” rate of growth coincided with the “natural” rate of growth. We are arguing that the Matthews effect may provide a mechanism to equilibrate the two.
totally approaching zero, while maintaining equilibrium $k = c/e$. Even less satisfactory, of course, is the alternative case where $g = h > n$ at the equilibrium $k$, which implies that the employment rate $e$ rises exponentially, in order to keep up with $c$.

The importance of the Matthews effect can also be ascertained from the stability condition. This condition is

$$\frac{dc}{dc} = \frac{dh}{dc} = \frac{dg}{dc} = \frac{(g_k + g_m')h_r}{e\lambda} < 0. \tag{4.5}$$

Note that this condition can only be satisfied if $h_r \neq 0$, and, more specifically, if the Malinvaud profitability effect on investment dominates the cost-minimization effect (such that $g_m < 0$), as we have been assuming, then we need $h_r > 0$. Hence, the Matthews effect is critical to both the existence and stability of steady state.

At the risk of oversimplifying, we offer the following verbalization of this result. Consider a rise in the capital/population ratio, $c$, (e.g. by a rise in $K$). This raises employment, but the multiplier is dampened by the Matthews effect on savings, $h_r > 0$. Consequently, $K$ rises proportionately more than $L$, so $k$ rises. This depresses investment ($h_i < 0$), so $c$ falls, giving us stability.

The exogenous effects on the steady state of the system are:

$$\frac{de^*}{dA} = -\frac{(h_i + h_m m')g_i}{|J|} > 0, \quad \frac{dw^*}{dA} = \frac{m'h_r g_i}{|J|} > 0, \quad \frac{dk^*}{dA} = \frac{h_r g_i}{|J|} > 0,$$

$$\frac{de^*}{dT} = \frac{-h_i}{h_r} < 0, \quad \frac{dw^*}{dT} = 0, \quad \frac{dk^*}{dT} = 0,$$

$$\frac{de^*}{dn} = -\frac{(|J| - h_r)e}{k|J|} < 0, \quad \frac{dw^*}{dn} = \frac{-m'h_r}{|J|} < 0, \quad \frac{dk^*}{dn} = \frac{-h_r}{|J|} < 0,$$

where $|J^*| = -e|J| \frac{dc}{dc} > 0$, by the stability condition.

Fig. 1 depicts the system response to an improvement in animal spirits. In the short run, employment rises, reducing the capital/labor ratio, and, in turn, the marginal productivity of labor, all in the usual Keynesian fashion. In the steady state, the employment rate improves further, while capital-deepening reverses the initial decline in the marginal productivity of labor, ultimately raising the steady-state wage.
Fig. 2 shows the response to an increase in thrift. Employment declines in the short run, raising the capital/labor ratio and the marginal productivity of labor, all in accord with the Keynesian paradox of thrift. In the steady state, employment continues to decline, while the capital-intensity and steady-state wage return to their original levels.

As Fig. 4 shows, the system's responses to a change in the population growth rate are qualitatively the same as in the neoclassical model.

To summarize, this section has shown how to construct an integrated short-run and steady-state Keynesian model. The Matthews effect is critical here, providing for the existence and stability of steady state. We have also analyzed the system response to an improvement in animal spirits, with the salient results that the employment multiplier persists to the steady state and that the steady-state wage improves. This contrasts, of course, with the neoclassical model, where animal spirits play no role. Furthermore, the effects of thrift on our Keynesian steady state depart dramatically from the neoclassical case, by reducing employment and leaving the wage unchanged.

5. A Keynesian Model: Labor Supply Pricing

In this section, we drop the marginal productivity condition, and take the wage to be the supply price of labor:

\[ w = w(e) + \tilde{w}, \quad w'(e) > 0, \]

(5.1)

where, again, we allow for a shift factor, \( \tilde{w} \), to raise the supply price (or, equivalently, reduce the supply of labor, for any given wage). This formulation allows the wage to vary procyclically, as argued by the critics of Keynes (1936). It also allows, as a special case, an exogenous wage, \( w = \tilde{w} \), as suggested by Keynes (1939). We will provide some discussion of this special case, for which \( w'(e) \equiv 0 \).

Short-run equilibrium is then given by

\[ h(k \equiv c/e, w(e) + \tilde{w}, e; T) = g(k \equiv c/e, w(e) + \tilde{w}; A). \]

(5.2)

The comparative static derivatives are:
\[
\frac{de}{dA} = \frac{g_k}{|J|} > 0, \quad \frac{de}{dc} = \frac{(g_k - h_k)}{e|J|} > 0, \\
\frac{de}{dT} = \frac{-h_r}{|J|} < 0, \quad \frac{de}{d\tilde{w}} = \frac{(g_w - h_w)}{|J|} \geq 0, \\
\frac{dk}{dA} = \frac{-kg_k}{e|J|} < 0, \quad \frac{dk}{dc} = \frac{h_r + (h_w - g_w) w'}{e|J|} \geq 0, \\
\frac{dk}{dT} = \frac{kh_1}{|J|} > 0, \quad \frac{dk}{d\tilde{w}} = \frac{k(h_w - g_w)}{e|J|} \geq 0, \\
\frac{dw}{dA} = \frac{w'g_k}{|J|} > 0, \quad \frac{dw}{dc} = \frac{(g_k - h_k) w'}{e|J|} > 0, \\
\frac{dw}{dT} = \frac{-w' h_1}{|J|} < 0, \quad \frac{dw}{d\tilde{w}} = \frac{h_c + (g_k - h_k) k/e}{|J|} > 0.
\]

where \(|J| = h_r + (g_k - h_k) k/e + (h_w - g_w) w' > 0|.

Note that the only difference in \(|J|\) from the previous model is that the third term represents the wage effects of moving up the supply curve now, rather than moving down the marginal productivity curve. While the sign of this term is still ambiguous, we again assume that it does not outweigh the other two terms, which are assumed to be positive. That is, we again assume that the Keynesian multiplier is positive (i.e. not explosive).

If steady state exists, the configuration for \(e^*\) and \(k^*\) is given by

\[
g(k^*, w(e^*) + \tilde{w}; A) = n. \tag{5.3}
\]

\[
h(k^*, w(e^*) + \tilde{w}, e^*; T) = n. \tag{5.4}
\]

The configuration is stable if

\[
d\tilde{c}/dc = d\tilde{h}/dc = dg/dc = 0.
\]

The parenthetical term is of ambiguous sign, but we assume that the first term, governed again by the Matthews effect, ensures stability. (Note that in the special case of the exogenous wage, \(w' = 0\), stability is governed solely by the Matthews effect, as in the previous section.)
The steady-state effects are

\[
\begin{align*}
\frac{de^*}{dT} &= \frac{g_k h_t}{|J^*|} < 0, \\
\frac{dk^*}{dT} &= \frac{h_{w'} + h_{c}}{|J^*|} \geq 0, \\
\frac{dw^*}{dT} &= \frac{w' h_{c}}{|J^*|} > 0,
\end{align*}
\]

where \( |J^*| = -e|J| \frac{dc}{dc} > 0 \), by the stability condition.

As Fig. 1 shows, a rise in animal spirits again increases employment both in the short run and in the steady state (though we cannot tell which multiplier is greater, as the hatched region indicates). Unlike the marginal productivity model, of course, the wage behaves procyclically as we move up the labor supply curve. As in the previous model, capital-intensity declines in the short run, as employment increases, but this may or may not be reversed in steady state.

Fig. 1. Improvement in animal spirits
(NA denotes “Not Applicable”)
Fig. 2 depicts an increase in thrift. Again, employment falls in both the short run and the steady state (though we cannot tell which decline is greater). Again, the wage moves procyclically, falling as we move back down the labor supply curve. Steady-state capital-intensity will rise if there is any slope at all to the labor supply curve (unlike the previous model), though it need not rise as much as it does in the short run.

![Fig. 2. Increase in thrift](image)

Fig. 3 considers an exogenous increase in the supply price of labor. The short-run effect on employment is ambiguous: it depends on whether savings or investment is more adversely affected by a rise in the wage. If investment is particularly depressed, as by Malinvaud's profitability mechanism, then so are output and employment, by the Keynesian multiplier; conversely, if savings are more depressed, by the classical Cantabrigian distribution effect, then output and employment are stimulated by a rise in the wage, following the paradox of thrift. This ambiguity persists to the steady state as well. The wage itself rises unambiguously in both the short run and the steady state (though we cannot be sure which is greater). The capital-intensity may rise or fall in the short run (due to the ambiguity in employment), but it must fall in the steady state.

Fig. 4 indicates that a change in the population growth rate affects the system much the same as in the previous models. There is, however, some ambiguity introduced regarding the behavior of capital-intensity.

To summarize, this section has shown that a Keynesian model can be equally well constructed using the labor supply condition
as it could using the marginal productivity condition. Unemployment, of course, is interpreted here as "voluntary", but it still responds to aggregate demand (e.g. "animal spirits"), which is the central theme of Keynesian economics. The choice between the two labor market assumptions is largely a distributional question, somewhat peripheral to this theme.

More specifically, we have shown that the employment responses to shifts in animal spirits and thrift are qualitatively robust with respect to the labor market assumption, as both Keynesian models contrast starkly with the neoclassical model. Perhaps the other noteworthy result of this section is that a rise in the supply price of labor does indeed raise the steady-state wage here, unlike the neoclassical model.
6. Conclusion

Our main contributions in this paper may be summarized as follows:

(1) It is possible to construct integrated short-run and steady-state Keynesian models. In doing so, however, the Matthews effect plays an important role in securing the existence and stability of steady state, quite unlike the neoclassical model.

In the neoclassical model, the Matthews effect is destabilizing through the following mechanism: if the growth rate of capital exceeds that of the population \( (h > n) \), raising \( c \), then labor demand shifts out more than labor supply, raising the wage and the employment rate. If this raises the propensity to save, by the Matthews effect, then this tends to further raise the growth rate of capital, which is destabilizing.

In Keynesian models, when the growth rate of capital exceeds that of the population \( (g = h > n) \), raising \( c \), this also raises the employment rate, through the Keynesian mechanism: savings falls short of \textit{ex ante} investment, leading to a multiplier expansion. However, the multiplier is dampened by the Matthews effect, so the capital/output ratio rises. This, in turn, dampens investment, slowing the growth of capital, which is stabilizing.

(2) In our Keynesian models, the employment multiplier with respect to animal spirits persists from the short run into the steady state. Furthermore, animal spirits raise the steady-state wage regardless of the labor market assumption. These results differ markedly from the neoclassical model, where animal spirits play no role.

(3) In our Keynesian models, thrift reduces employment in both the short run and the steady state, and does not increase the steady-state wage, regardless of the labor market assumptions. These steady-state results are quite the opposite of the neoclassical model.

(4) An exogenous increase in the labor supply price raises the steady-state wage in the applicable Keynesian model, and reduces it in the neoclassical model. The response of the employment rate, which is the main concern of Malinvaud and other observers of the European situation, is adverse in the neoclassical model, but in the Keynesian model it is only adverse if the Malinvaud profitability effect dominates both the cost-minimization effect and the classical/Cantabrigian distribution effect.
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